Iterating Maps.

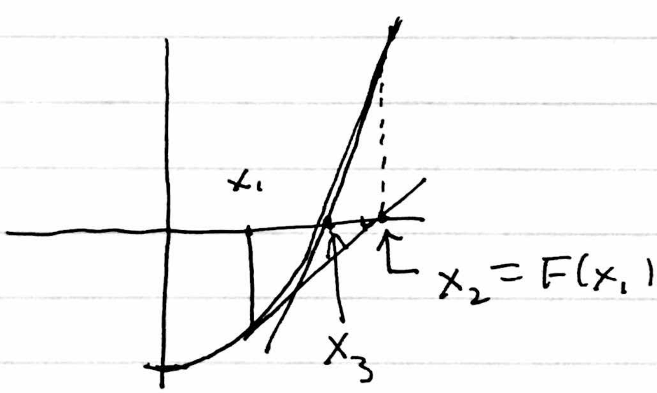
last time: ~~we~~ we saw that
if we 'seed' the map ~~with~~
$x \longmapsto F(x) = \frac{1}{2}\left(x + \frac{2}{x}\right)$

with some value $x_1 > 0$ & iterate:

$x_2 = F(x_1),\ x_3 = F(x_2),\ x_4 = F(x_3), \ldots$

then the sequence of points $x_i \in \mathbb{R}$ converge to $\sqrt{2}$.

We derived F as follows:



pretend linear approx to $x^2 + 2 = f(x)$ is exact at x_1 to find x_2.

(General formula:
$$F(x) = x - \frac{f(x)}{f'(x)}.$$
)

Instead we can iterate a general map (= function).

Famous example.

"logistic map" , cf May article; text §1.6.

$$F(x) = rx(1-x)$$

$\uparrow$ ~~pa~~parameter. indep of r.

used in population modelling, recall

$$\frac{dN(t)}{dt} = rN(t)(1-N(t))$$

which inspired it

cf May article

who instead writes:

$$X_{t+1} = F(X_t)$$

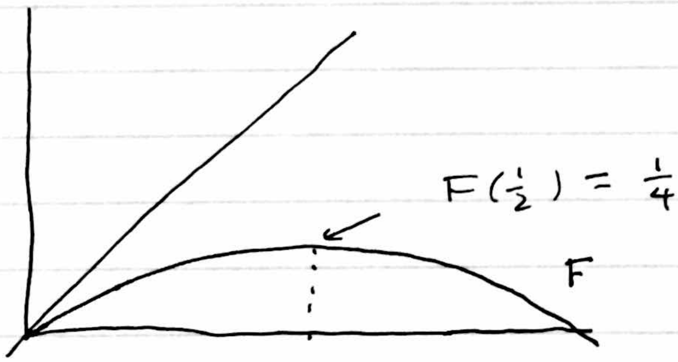
t = discrete time, $t = 0, 1, 2, 3$
= "generations"

$X \mapsto F(X)$ population model,

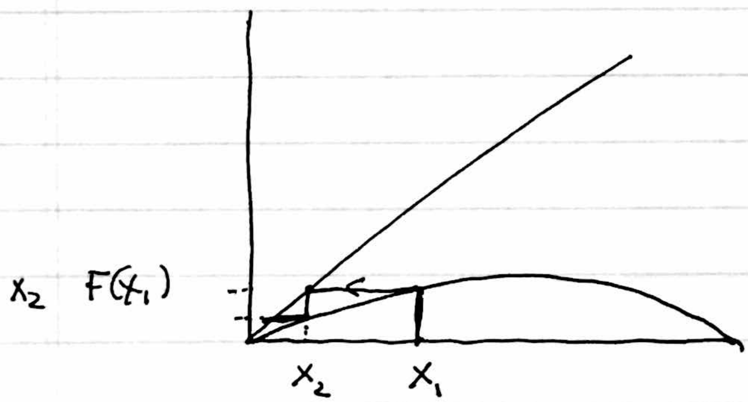
X = population density.

lets see: $r < 1$, eg $r = \frac{1}{2}$.

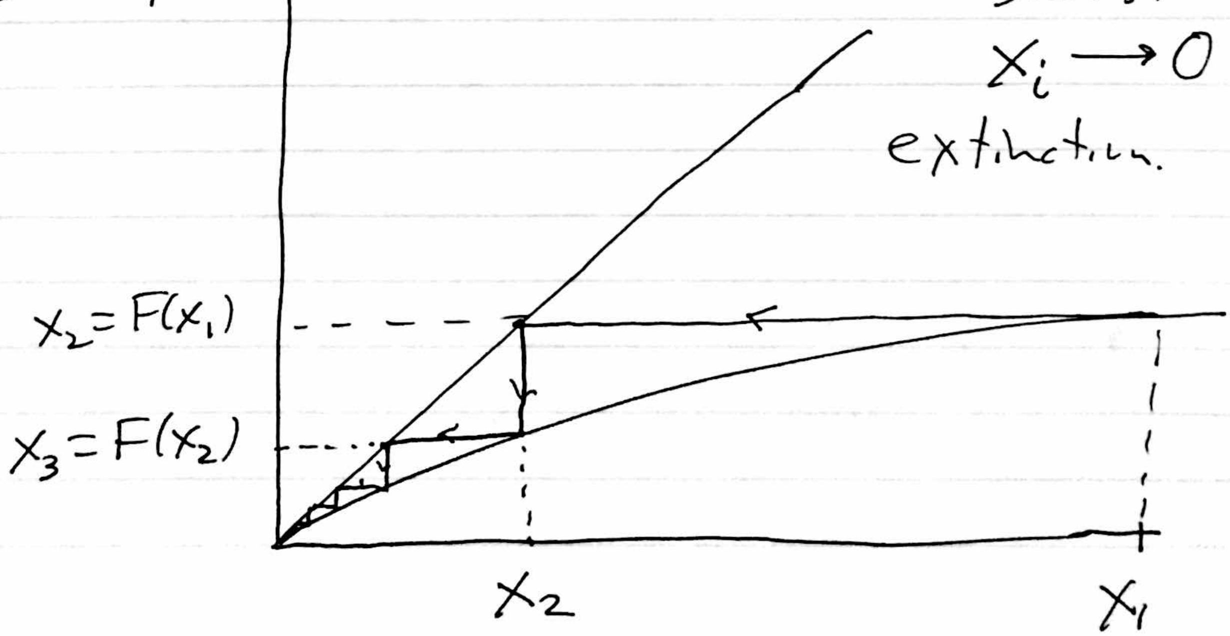
$F(x) = \frac{1}{2} x(1-x)$



to see what happens:
"cobweb"

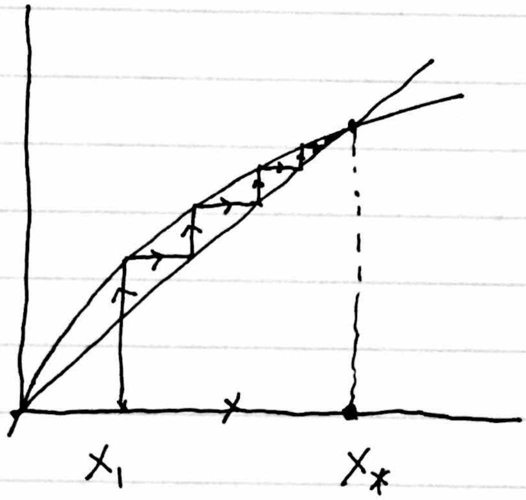


Blow up.



Picture shows:
$x_i \longrightarrow 0$!
extinction.

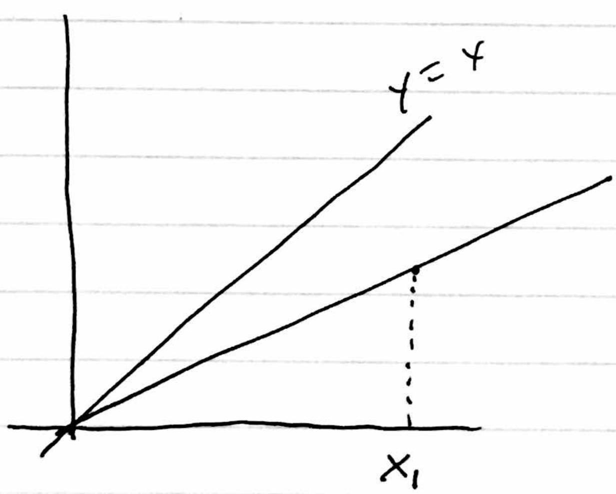
$r > 1$ but not much.



$x_i \longrightarrow x_*$ a new fixed point > 0

"stable population"

Qu.



$y = \frac{2}{3} x$.

so $F(x) = \frac{2}{3} x$.

what happens to whit?
Draw cobweb diagram.
where does x_i go?

Ques ~~F~~ iteration whit for
$F(x) = 2x$.

where does x_n go?
w/ $x_{n+1} = F(x_n)$.