

More Practice Final problems, WINTER 2020

1. $F(x) = rx - x^4$ is to be viewed as a mapping on $[0, 1]$. The parameter r is positive. Note $F(0) = 0$ so that 0 is a fixed point for all values of r .

a) As r increases from 0, beyond which value of r does a new fixed point x_* appear for F , that is, a fixed point with $x_* > 0$?

b) Let r continue to increase beyond the value of r found in (a). Beyond which value of r does the new fixed point x_* become unstable?

Similar problems to the above: end of chapter 4.5 of text.

2. Simplify $e^{2\ln(x^2+2)}$

3. Suppose that p is a polynomial of degree 10 and that $p(3) = 0$, $p'(3) = 2$

a) Compute the derivative of $f(x) = e^{p(x)}$ at $x = 3$.

b) Compute the derivative of $g(x) = \ln(1 + p(x))$ at $x = 3$.

4. Review the binomial expansion for $f(x) = (1 + x)^\alpha$ where α is a constant exponent.

a) Use the binomial expansion, or the linear approximation, to compute $1002^{4/3}$. [Note: $1000 = 10^3$.]

5. Read the last two pages of the May article, especially the conclusion. Discuss with a friend or classmate.

6. What is the maximum value and minimum value of the derivative of $f(x) = \cos(7x)$? Of $\sin(7x)$?

Tangent line problems

6. Find an equation for the tangent line to the graphs of the following functions at the given points

a) $\sin(x)$. (i) $x = 0$, (ii) $x = \pi/3$, (iii) $x = \pi/2$

b) $\cos(x)$. (i) $x = 0$, (ii) $x = \pi/3$, (iii) $x = \pi/2$

c) $e^{3x} - 1$ (i) $x = 0$, (ii) $x = -1$

d) $p(x) = 1 + 2x + 3x^2 + 4x^3 + 5x^4 + \dots + 100x^{99}$, (i) just at $x = 0$.

Optimization Problems

The distance between a point $P_0 = (x_0, y_0)$ and a line ℓ in the plane is the answer to an optimization problem: minimize the distance $\|P_0 - P\|$ between P and a variable point $P = (x, y) \in \ell$ and P_0 .

Understand this as a calculus problem.

Know the formula for the distance: $\|P_0 - P\| = \sqrt{(x - x_0)^2 + (y - y_0)^2}$.

Understand that minimizing the distance is the same as minimizing its square.

Know how to compute the minimizing point $P_* \in \ell$ and the value of the distance.

Example problems:

What is the distance between the origin $P_0 = (0, 0)$ and

a) the line $x = 1$?

b) the line $y = 1$?

c) the line $x + y = 1$?

c) The distance between the point $P_0 = (0, 1)$ and the line $y = mx$ where m is a constant slope.

Finally, understand the basic geometry of this problem: Why is it that the segment P_0P_* must be perpendicular to the line ℓ at the minimization point $P_* \in \ell$.