QUESTION 1. A certain function f having continuous derivatives satisfies $f(0) = 0, f(10) = 0$ and $f'(0) = -5$. **TRUE or FALSE?** There is an x between 0 and 10 for which $f'(x) = -1/2$.

ANSWER: TRUE.

SOLUTION: Draw the line joining $(0, f(0))$ to $(10, f(10))$ and observe that the slope of this line is 0. By the MEAN value theorem there must be a point between 0 and 10 such that $f'(x_*) = 0$. Now $f'(x)$ takes the value -5 at $x = 0$ and 0 at c_*, and we are told $f'(x)$ is continuous. Apply the INTERMEDIATE value theorem to $f'(x)$ to get tht $f'(x)$ takes on every value between -5 and 0 as x travels between 0 and x_*. Since $-1/2$ is between -5 and 0 get that there is some x_{**} between 0 and x_* with $f'(x_{**}) = -1/2$. Finally, note that $0 < x_{**} < 10$.

QUESTION 6.

TRUE or FALSE?

$$\frac{1}{1 + \frac{2}{\frac{1}{1+\frac{3}{4}}}} < 1/3$$

ANSWER: TRUE

SOLUTION 1 : The fraction on the right hand side is similar to $\frac{1}{1+2} = \frac{1}{3}$. The strange fraction is of the precise form $\frac{1}{1+\frac{2}{a}}$ where $a = \frac{1}{1+\frac{3}{4}}$. If $a > 1$ then $2/a < 2$ so $1 + 2/a < 3$ and we have that $\frac{1}{1+\frac{2}{a}} > 1/3$. On the other hand, if $a < 1$ then $2/a > 2$ and so $1 + 2/a > 3$ and our final fraction $\frac{1}{1+\frac{2}{a}} < 1/3$. Well $1 + 3/4 > 1$ so in our case $a = 1/(1 + 3/4) < 1$ and we are in the second eventuality: the strange fraction is less than 1/3.

SOLUTION 2. $1 + 3/4 = 7/4$. So $1/(1 + 3/4) = 4/7$. So $\frac{2}{\frac{1}{1+\frac{3}{4}}} = 2/(4/7) = 14/4 = 7/2$. Then $1 + \frac{2}{\frac{1}{1+\frac{3}{4}}} = 2/2 + 7/2 = 9/2$. Finally, $\frac{1}{1+\frac{2}{\frac{1}{1+\frac{3}{4}}}} = 1/(9/2) = 2/9 < 3/9 = 1/3$.