CONCEPT CHECK

1. Explain the difference between an absolute maximum and a local maximum. Illustrate with a sketch.
2. (a) What does the Extreme Value Theorem say?
   (b) Explain how the Closed Interval Method works.
3. (a) State Fermat's Theorem.
   (b) Define a critical number of f.
4. State the Mean Value Theorem and give a geometric interpretation.
5. (a) State the Increasing/Decreasing Test.
   (b) What does it mean to say that f is concave upward on an interval I?
   (c) State the Concavity Test.
   (d) What are inflection points? How do you find them?
6. (a) State the First Derivative Test.
   (b) State the Second Derivative Test.
   (c) What are the relative advantages and disadvantages of these tests?
7. (a) What does l'Hospital's Rule say?
   (b) How can you use l'Hospital's Rule if you have a product $f(x)g(x)$ where $f(x) \to 0$ and $g(x) \to \infty$ as $x \to a$?
   (c) How can you use l'Hospital's Rule if you have a difference $f(x) - g(x)$ where $f(x) \to \infty$ and $g(x) \to \infty$ as $x \to a$?
8. (a) What is an equilibrium of the recursive sequence $x_{t+1} = f(x_t)$?
   (b) What is a stable equilibrium? An unstable equilibrium?
   (c) State the Stability Criterion.
9. (a) What is an antiderivative of a function f?
   (b) Suppose F_1 and F_2 are both antiderivatives of f on an interval I. How are F_1 and F_2 related?

Answers to the Concept Check can be found on the back endpapers.

TRUE-FALSE QUIZ

Determine whether the statement is true or false. If it is true, explain why. If it is false, explain why or give an example that disproves the statement.

1. If $f'(c) = 0$, then f has a local maximum or minimum at c.
2. If f has an absolute minimum value at c, then $f'(c) = 0$.
3. If f is continuous on (a, b), then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in (a, b).
4. If f is differentiable and $f(-1) = f(1)$, then there is a number c such that $|c| < 1$ and $f'(c) = 0$.
5. If $f'(x) < 0$ for $1 < x < 6$, then f is decreasing on $(1, 6)$.
6. If $f''(2) = 0$, then $(2, f(2))$ is an inflection point of the curve $y = f(x)$.
7. If $f'(x) = g'(x)$ for $0 < x < 1$, then $f(x) = g(x)$ for $0 < x < 1$.
8. There exists a function f such that $f(1) = -2$, $f(3) = 0$, and $f'(x) > 1$ for all x.
9. There exists a function f such that $f(x) > 0$, $f'(x) < 0$, and $f''(x) > 0$ for all x.
10. There exists a function f such that $f(x) < 0$, $f'(x) < 0$, and $f''(x) > 0$ for all x.
11. If f and g are increasing on an interval I, then $f + g$ is increasing on I.
12. If f and g are increasing on an interval I, then $f - g$ is increasing on I.
13. If f and g are increasing on an interval I, then fg is increasing on I.
14. If f and g are positive increasing functions on an interval I, then fg is increasing on I.
15. If f is increasing and $f(x) > 0$ on I, then $g(x) = 1/f(x)$ is decreasing on I.
16. If f is even, then f' is even.
17. If f is periodic, then f' is periodic.
18. $\displaystyle\lim_{x \to 0} \frac{x}{e^x} = 1$

EXERCISES

1–6 Find the local and absolute extreme values of the function on the given interval.

1. $f(x) = x^3 - 6x^2 + 9x + 1$, $[2, 4]$
2. $f(x) = x\sqrt{1 - x}$, $[-1, 1]$
3. $f(x) = \dfrac{3x - 4}{x^2 + 1}$, $[-2, 2]$
4. $f(x) = (x^2 + 2x)^3$, $[-2, 1]$
5. $f(x) = x + \sin 2x$, $[0, \pi]$
6. $f(x) = (\ln x)/x^2$, $[1, 3]$

7–14
(a) Find the vertical and horizontal asymptotes, if any.
(b) Find the intervals of increase or decrease.
(c) Find the local maximum and minimum values.
(d) Find the intervals of concavity and the inflection points.
(e) Use the information from parts (a)–(d) to sketch the graph of f. Check your work with a graphing device.

7. $f(x) = 2 - 2x - x^3$
8. $f(x) = x^4 + 4x^3$
9. $f(x) = x + \sqrt{1 - x}$
10. $f(x) = \dfrac{1}{1 - x^2}$
11. $y = \sin^2 x - 2\cos x$
12. $y = e^{2x - x^2}$
13. $y = e^x + e^{-3x}$
14. $y = \ln(x^2 - 1)$

15. **Antibiotic pharmacokinetics** A model for the concentration of an antibiotic drug in the bloodstream t hours after the administration of the drug is

$$C(t) = 2.5(e^{-0.3t} - e^{-0.7t})$$

where C is measured in μg/mL.
(a) At what time does the concentration have its maximum value? What is the maximum value?
(b) At what time does the inflection point occur? What is the significance of the inflection point?

16. **Drug pharmacokinetics** Another model for the concentration of a drug in the bloodstream is

$$C(t) = 0.5t^2 e^{-0.6t}$$

where t is measured in hours and C is measured in μg/mL.
(a) At what time does the concentration have its largest value? What is the largest value?
(b) How many inflection points are there? At what times do they occur? What is the significance of each inflection point?
(c) Compare the graphs of this concentration function and the one in Exercise 15. How are the graphs similar? How are they different?

17. **Population bound** Suppose that an initial population size is 300 individuals and the population grows at a rate of at most 120 individuals per week. What can you say about the population size after five weeks?

18. Sketch the graph of a function that satisfies the following conditions:
$f(0) = 0$, f is continuous and even,
$f'(x) = 2x$ if $0 < x < 1$, $f'(x) = -1$ if $1 < x < 3$,
$f'(x) = 1$ if $x > 3$

19–25 Evaluate the limit.

19. $\displaystyle\lim_{x \to 0} \frac{\tan \pi x}{\ln(1 + x)}$
20. $\displaystyle\lim_{x \to 0} \frac{1 - \cos x}{x^2 + x}$
21. $\displaystyle\lim_{x \to 0} \frac{e^{4x} - 1 - 4x}{x^2}$
22. $\displaystyle\lim_{x \to \infty} \frac{e^{4x} - 1 - 4x}{x^2}$
23. $\displaystyle\lim_{x \to \infty} x^3 e^{-x}$
24. $\displaystyle\lim_{x \to 0^+} x^2 \ln x$
25. $\displaystyle\lim_{x \to 1^+} \left(\frac{x}{x - 1} - \frac{1}{\ln x}\right)$

26. Rank the functions in order of how quickly they grow as $x \to \infty$.

$y = \sqrt[4]{x}$ $\quad y = \ln(10x)$ $\quad y = 10^x$ $\quad y = \sqrt{1 + e^x}$

27. Find two positive integers such that the sum of the first number and four times the second number is 1000 and the product of the numbers is as large as possible.

28. Find the point on the hyperbola $xy = 8$ that is closest to the point $(3, 0)$.

29. The velocity of a wave of length L in deep water is

$$v = K\sqrt{\frac{L}{C} + \frac{C}{L}}$$

where K and C are known positive constants. What is the length of the wave that gives the minimum velocity?

30. The **Ricker model** for population growth is a discrete-time model of the form

$$n_{t+1} = cn_t e^{-\lambda n_t}$$

For the constants $c = 2$ and $\lambda = 3$, the model is $n_{t+1} = f(n_t)$, where the updating function is

$$f(n) = 2ne^{-3n}$$

Find the largest value of f and interpret it.

31. **Drug resistance evolution** A simple model for the spread of drug resistance is given by $\Delta p = p(1 - p)s$, where s is a measure of the reproductive advantage of the drug resistance gene in the presence of drugs, p is the

frequency of the drug resistance gene in the population, and Δp is the change in the frequency of the drug resistance gene in the population after one year. Notice that the amount of change in the frequency Δp differs depending on the gene's current frequency p. What current frequency makes the rate of evolution Δp the largest?

32. The **thermic effect of food** (TEF) is the increase in metabolic rate after a meal. Researchers used the functions

$$f(t) = 175.9te^{-t/1.3} \qquad g(t) = 113.6te^{-t/1.85}$$

to model the TEF (measured in kJ/h) for a lean person and an obese person, respectively.

(a) Find the maximum value of the TEF for both individuals.

(b) Graph the TEF functions for both individuals. Describe how the graphs are similar and how they differ.

Source: Adapted from G. Reed et al., "Measuring the Thermic Effects of Food." *American Journal of Clinical Nutrition* 63 (1996): 164–69.

33–34 Find the equilibria of the difference equation and classify them as stable or unstable.

33. $x_{t+1} = \dfrac{4x_t}{1 + 5x_t}$ **34.** $x_{t+1} = 5x_t e^{-4x_t}$

35. Find the equilibria of the difference equation

$$x_{t+1} = \frac{6x_t^2}{x_t^2 + 8}$$

and classify them as stable or unstable. Use cobwebbing to evaluate $\lim_{t\to\infty} x_t$ for $x_0 = 1$ and $x_0 = 3$.

36. Let $f(x) = 1.07x + \sin x$, $0 \leqslant x \leqslant 11$. How many equilibria does the recursion $x_{t+1} = f(x_t)$ have? Estimate their values and explain why they are stable or unstable.

37–40 Find the most general antiderivative of the function.

37. $f(x) = \sin x + \sec x \tan x$, $0 \leqslant x < \pi/2$

38. $g(t) = (1 + t)/\sqrt{t}$

39. $q(t) = 2 + (t + 1)(t^2 - 1)$

40. $w(\theta) = 2\theta - 3\cos\theta$

41–42 Solve the initial-value problem.

41. $\dfrac{dy}{dt} = 1 - e^{\pi t}$, $y = 0$ when $t = 0$

42. $\dfrac{dr}{dt} = \dfrac{4}{1 + t^2}$, $r(1) = 2$

43–44 Find $f(x)$.

43. $f''(x) = 1 - 6x + 48x^2$, $f(0) = 1$, $f'(0) = 2$

44. $f''(x) = 2x^3 + 3x^2 - 4x + 5$, $f(0) = 2$, $f(1) = 0$

45. A particle moves in a straight line with acceleration $a(t) = \sin t + 3\cos t$, initial displacement $s(0) = 0$, and initial velocity $v(0) = 2$. Find its position function $s(t)$.

46. Sketch the graph of a continuous, even function f such tha $f(0) = 0$, $f'(x) = 2x$ if $0 < x < 1$, $f'(x) = -1$ if $1 < x < 3$, and $f'(x) = 1$ if $x > 3$.

47. If a rectangle has its base on the x-axis and two vertices or the curve $y = e^{-x^2}$, show that the rectangle has the largest possible area when the two vertices are at the points of inflection of the curve.