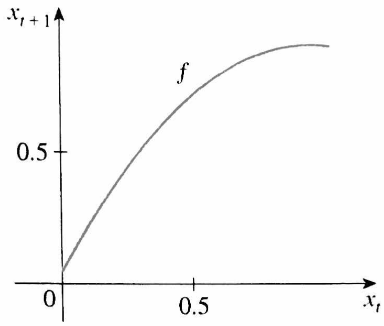
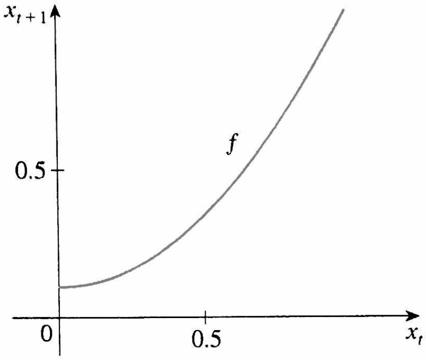
EXERCISES 4.5

1–4 The graph of the function f for a recursive sequence $x_{t+1} = f(x_t)$ is shown. Estimate the equilibria and classify them as stable or unstable. Confirm your answer by cobwebbing.

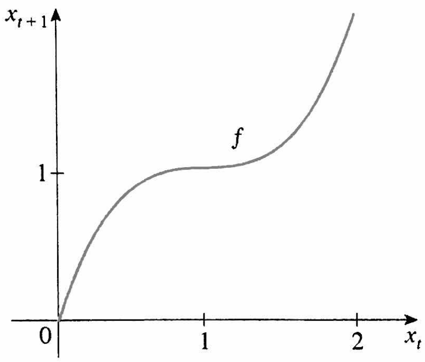
1.



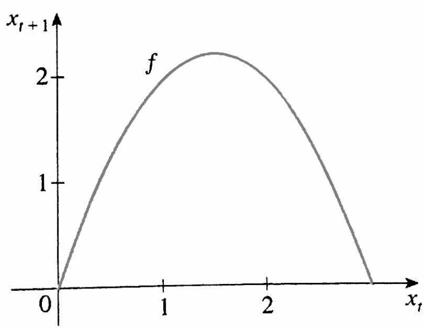
2.



3.



4.



5–10 Find the equilibria of the difference equation and classify them as stable or unstable.

5. $x_{t+1} = \frac{1}{2}x_t^2$

6. $x_{t+1} = 1 - x_t^2$

7. $x_{t+1} = \dfrac{x_t}{0.2 + x_t}$

8. $x_{t+1} = \dfrac{3x_t}{1 + x_t}$

9. $x_{t+1} = 10x_t e^{-2x_t}$

10. $x_{t+1} = x_t^3 - 3x_t^2$ -

11–12 Find the equilibria of the difference equation and classify them as stable or unstable. Use cobwebbing to fi $\lim_{t\to\infty} x_t$ for the given initial values.

11. $x_{t+1} = \dfrac{4x_t^2}{x_t^2 + 3}$, $x_0 = 0.5$, $x_0 = 2$

12. $x_{t+1} = \dfrac{7x_t^2}{x_t^2 + 10}$, $x_0 = 1$, $x_0 = 3$

13–14 Find the equilibria of the difference equation. Det mine the values of c for which each equilibrium is stable

13. $x_{t+1} = \dfrac{cx_t}{1 + x_t}$

14. $x_{t+1} = \dfrac{x_t}{c + x_t}$

15. Drug pharmacokinetics A patient takes 200 mg c drug at the same time every day. Just before each tab taken, 10% of the drug remains in the body.
(a) If Q_n is the quantity of the drug in the body just a the nth tablet is taken, write a difference equatio expressing Q_{n+1} in terms of Q_n.
(b) Find the equilibria of the equation in part (a).
(c) Draw a cobwebbing diagram for the equation.

16. Drug pharmacokinetics A patient is injected with drug every 8 hours. Immediately before each injectic concentration of the drug has been reduced by 40% a new dose increases the concentration by 1.2 mg/mL
(a) If Q_n is the concentration of the drug in the body after the nth injection is given, write a difference tion expressing Q_{n+1} in terms of Q_n.
(b) Find the equilibria of the equation in part (a).
(c) Draw a cobwebbing diagram for the equation.

17–20 Logistic difference equation Illustrate the resu Example 4 for the logistic difference equation by cobwet and by graphing the first ten terms of the sequence for the values of c and x_0.

17. $c = 0.8$, $x_0 = 0.6$

18. $c = 1.8$, $x_0 = 0.1$

19. $c = 2.7$, $x_0 = 0.1$

20. $c = 3.6$, $x_0 = 0.4$

21. Sustainable harvesting In Example 4.4.5 we looked at a model of sustainable harvesting, which can be formulated as a discrete-time model:

$$N_{t+1} = N_t + rN_t\left(1 - \frac{N_t}{K}\right) - hN_t$$

Find the equilibria and determine when each is stable.

22. Heart excitation A simple model for the time x_t it takes for an electrical impulse in the heart to travel through the atrioventricular node of the heart is

$$x_{t+1} = \frac{375}{x_t - 90} + 100 \qquad x_t > 90$$

(a) Find the relevant equilibrium and determine when it is stable.
(b) Draw a cobwebbing diagram.

Source: Adapted from D. Kaplan et al., *Understanding Nonlinear Dynamics* (New York: Springer-Verlag, 1995).

23. Species discovery curves A common assumption is that the rate of discovery of new species is proportional to the fraction of currently undiscovered species. If d_t is the fraction of species discovered by time t, a recursion equation describing this process is

$$d_{t+1} = d_t + a(1 - d_t)$$

where a is a constant representing the discovery rate and satisfies $0 < a < 1$. Find the equilibria and determine the stability.

24. Drug resistance in malaria In the project on page 78 we developed the following recursion equation for the spread of a gene for drug resistance in malaria:

$$p_{t+1} = \frac{p_t^2 W_{RR} + p_t(1 - p_t)W_{RS}}{p_t^2 W_{RR} + 2p_t(1 - p_t)W_{RS} + (1 - p_t)^2 W_{SS}}$$

where W_{RR}, W_{RS}, and W_{SS} are constants representing the probability of survival of the three genotypes. In fact this model applies to the evolutionary dynamics of any gene in a population of diploid individuals.
(a) Find the equilibria of the model in terms of the constants.
(b) Suppose that $W_{RR} = \frac{3}{4}$, $W_{RS} = \frac{1}{2}$, and $W_{SS} = \frac{1}{4}$. Determine the stability of each equilibrium (provided it lies between 0 and 1). Plot the cobwebbing diagram and interpret your results.
(c) Suppose that $W_{RR} = \frac{1}{2}$, $W_{RS} = \frac{3}{4}$, and $W_{SS} = \frac{1}{4}$. Determine the stability of each equilibrium. Plot the cobwebbing diagram and interpret your results.

25. Blood cell production A simple model of blood cell production is given by

$$R_{t+1} = R_t(1 - d) + F(R_t)$$

where d is the fraction of red blood cells that die from one day to the next and $F(x)$ is a function specifying the number of new cells produced in a day, given that the current number is x. Find the equilibria and determine the stability in each case.
(a) $F(x) = \theta(K - x)$, where θ and K are positive constants
(b) $F(x) = \dfrac{ax}{b + x^2}$, where a and b are positive constants and $a > bd$

Source: Adapted from N. Mideo et al., "Understanding and Predicting Strain-Specific Patterns of Pathogenesis in the Rodent Malaria *Plasmodium chabaudi*," *The American Naturalist* 172 (2008): E214–E328.

4.6 Antiderivatives

Suppose you know the rate at which a bacteria population is increasing and want to know the size of the population at some future time. Or suppose you know the rate of decrease of your blood alcohol concentration and want to know your BAC an hour from now. In each case, the problem is to find a function F whose derivative is a known function f. If such a function F exists, it is called an *antiderivative* of f.

> **Definition** A function F is called an **antiderivative** of f on an interval I if $F'(x) = f(x)$ for all x in I.

For instance, let $f(x) = x^2$. It isn't difficult to discover an antiderivative of f if we keep the Power Rule in mind. In fact, if $F(x) = \frac{1}{3}x^3$, then $F'(x) = x^2 = f(x)$. But the function $G(x) = \frac{1}{3}x^3 + 100$ also satisfies $G'(x) = x^2$. Therefore both F and G are antiderivatives of f. Indeed, any function of the form $H(x) = \frac{1}{3}x^3 + C$, where C is a constant, is an antiderivative of f. The following theorem says that f has no other antiderivative. A proof of Theorem 1, using the Mean Value Theorem, is outlined in Exercise 46.