Then

$$T_1(x) = f(1) + f'(1)(x - 1) = x - 1$$

$$T_2(x) = T_1(x) + \frac{f''(1)}{2!}(x - 1)^2 = x - 1 - \tfrac{1}{2}(x - 1)^2$$

$$T_3(x) = T_2(x) + \frac{f'''(1)}{3!}(x - 1)^3 = x - 1 - \tfrac{1}{2}(x - 1)^2 + \tfrac{1}{3}(x - 1)^3$$

Figure 12 shows the graphs of these Taylor polynomials. Notice that these polynomial approximations are better when x is close to 1 and that each successive approximation is better than the preceding ones.

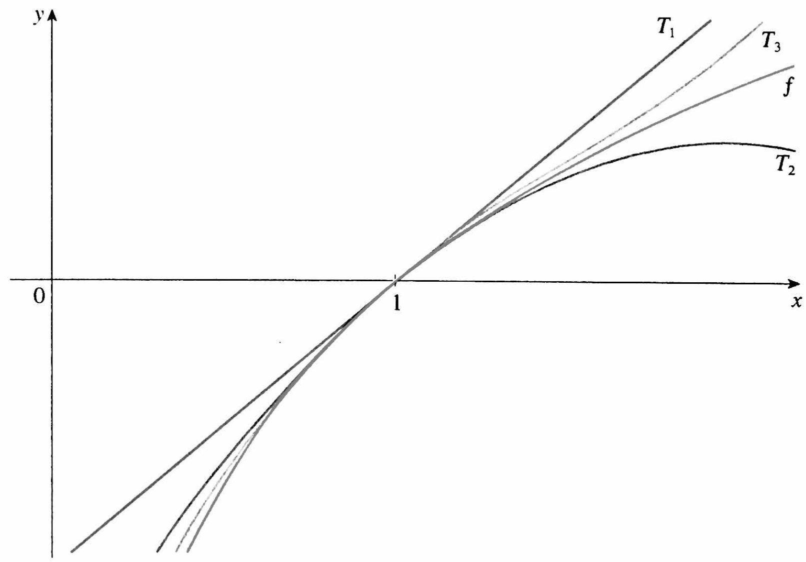


FIGURE 12

EXERCISES 3.8

1–4 Find the linearization $L(x)$ of the function at a.

1. $f(x) = x^4 + 3x^2, \quad a = -1$ **2.** $f(x) = \ln x, \quad a = 1$

3. $f(x) = \cos x, \quad a = \pi/2$ **4.** $f(x) = x^{3/4}, \quad a = 16$

5. Find the linear approximation of the function $f(x) = \sqrt{1 - x}$ at $a = 0$ and use it to approximate the numbers $\sqrt{0.9}$ and $\sqrt{0.99}$. Illustrate by graphing f and the tangent line.

6. Find the linear approximation of the function $g(x) = \sqrt[3]{1 + x}$ at $a = 0$ and use it to approximate the numbers $\sqrt[3]{0.95}$ and $\sqrt[3]{1.1}$. Illustrate by graphing g and the tangent line.

7–10 Verify the given linear approximation at $a = 0$. Then determine the values of x for which the linear approximation is accurate to within 0.1.

7. $\sqrt[3]{1 - x} \approx 1 - \frac{1}{3}x$ **8.** $\tan x \approx x$

9. $1/(1 + 2x)^4 \approx 1 - 8x$ **10.** $e^x \approx 1 + x$

11–12 Use a linear approximation to estimate the given numbe

11. $(2.001)^5$ **12.** $e^{-0.015}$

13–14 Explain, in terms of linear approximations, why the approximation is reasonable.

13. $\ln 1.05 \approx 0.05$ **14.** $(1.01)^6 \approx 1.06$

15. Insecticide resistance If the frequency of a gene for insecticide resistance is p (a constant), then its frequency i the next generation is given by the expression

$$f = \frac{p(1 + s)}{1 + sp}$$

where s is the reproductive advantage this gene has over th wild type in the presence of the insecticide. Often the selective advantage s is very small. Approximate the frequency in the next generation with a linear approximation, given that s is small.

16. Relative change in blood velocity Suppose $y = f(x)$ and x and y change by amounts Δx and Δy. A way of expressing a linear approximation is to write $\Delta y \approx f'(x)\,\Delta x$. The *relative change* in y is $\Delta y/y$.

A special case of Poiseuille's law of laminar flow (see Example 3.3.9) is that at the central axis of a blood vessel the velocity of the blood is related to the radius R of the vessel by an equation of the form $v = cR^2$. If the radius changes, how is the relative change in the blood velocity related to the relative change in the radius? If the radius is increased by 10%, what happens to the velocity?

17. Relative change in blood flow Another law of Poiseuille says that when blood flows along a blood vessel, the flux F (the volume of blood per unit time that flows past a given point) is proportional to the fourth power of the radius R of the blood vessel:

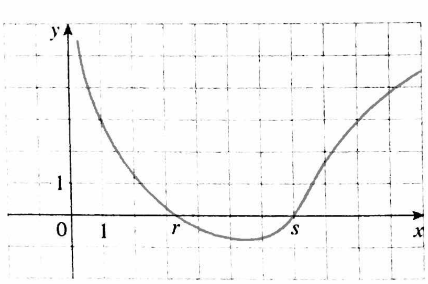
$$F = kR^4$$

(We will show why this is true in Section 6.3.) A partially clogged artery can be expanded by an operation called angioplasty, in which a balloon-tipped catheter is inflated inside the artery in order to widen it and restore the normal blood flow.

Show that the relative change in F is about four times the relative change in R. How will a 5% increase in the radius affect the flow of blood?

18. Volume and surface area of a tumor The diameter of a tumor was measured to be 19 mm. If the diameter increases by 1 mm, use linear approximations to estimate the relative changes in the volume $\left(V = \frac{4}{3}\pi r^3\right)$ and surface area $(S = 4\pi r^2)$.

19. The figure shows the graph of a function f. Suppose that Newton's method is used to approximate the root r of the equation $f(x) = 0$ with initial approximation $x_1 = 1$.
(a) Draw the tangent lines that are used to find x_2 and x_3, and estimate the numerical values of x_2 and x_3.
(b) Would $x_1 = 5$ be a better first approximation? Explain.



20. Follow the instructions for Exercise 19(a) but use $x_1 = 9$ as the starting approximation for finding the root s.

21–22 Use Newton's method with the specified initial approximation x_1 to find x_3, the third approximation to the root of the given equation. (Give your answer to four decimal places.)

21. $x^3 + 2x - 4 = 0, \quad x_1 = 1$

22. $\frac{1}{3}x^3 + \frac{1}{2}x^2 + 3 = 0, \quad x_1 = -3$

23–26 Use Newton's method to find all roots of the equation correct to six decimal places.

23. $x^4 = 1 + x$

24. $e^x = 3 - 2x$

25. $(x - 2)^2 = \ln x$

26. $\dfrac{1}{x} = 1 + x^3$

27–31 Use Newton's method to find all the roots of the equation correct to eight decimal places. Start by drawing a graph to find initial approximations.

27. $x^6 - x^5 - 6x^4 - x^2 + x + 10 = 0$

28. $x^2(4 - x^2) = \dfrac{4}{x^2 + 1}$

29. $x^2\sqrt{2 - x - x^2} = 1$

30. $3\sin(x^2) = 2x$

31. $4e^{-x^2}\sin x = x^2 - x + 1$

32. Infectious disease outbreak size If 99% of a population is initially uninfected and each initial infected person generates, on average, two new infections, then, according to the model we considered in Example 3.5.13,

$$0.99e^{-2A} = 1 - A$$

where A is the fraction of the population infected at the end of an outbreak. Use Newton's method to obtain an approximation (accurate to two decimal places) for the percentage of the population that is eventually infected.

33. Explain why Newton's method doesn't work for finding the root of the equation $x^3 - 3x + 6 = 0$ if the initial approximation is chosen to be $x_1 = 1$.

34. (a) Use Newton's method with $x_1 = 1$ to find the root of the equation $x^3 - x = 1$ correct to six decimal places.
(b) Solve the equation in part (a) using $x_1 = 0.6$ as the initial approximation.
(c) Solve the equation in part (a) using $x_1 = 0.57$. (You definitely need a programmable calculator for this part.)
(d) Graph $f(x) = x^3 - x - 1$ and its tangent lines at $x_1 = 1, 0.6$, and 0.57 to explain why Newton's method is so sensitive to the value of the initial approximation.

35–38 Find the Taylor polynomial of degree n centered at the number a.

35. $f(x) = e^x, \quad n = 3, \quad a = 0$