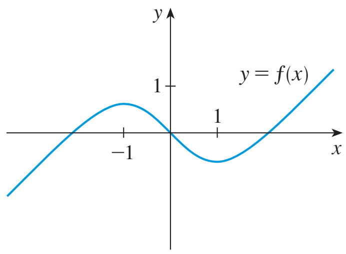


FIGURE 18

SOLUTION

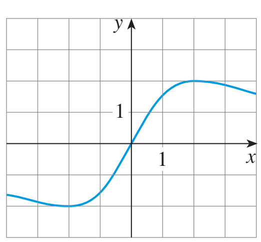
(a) We observe from Figure 17 that $f'(x)$ is negative when $-1 < x < 1$, so the original function f must be decreasing on the interval $(-1, 1)$. Similarly, $f'(x)$ is positive for $x < -1$ and for $x > 1$, so f is increasing on the intervals $(-\infty, -1)$ and $(1, \infty)$. Also note that, since $f'(-1) = 0$ and $f'(1) = 0$, the graph of f has horizontal tangents when $x = \pm 1$.

(b) We use the information from part (a), and the fact that the graph passes through the origin, to sketch a possible graph of f in Figure 18. Notice that $f'(0) = -1$, so we have drawn the curve $y = f(x)$ passing through the origin with a slope of -1. Notice also that $f'(x) \to 1$ as $x \to \pm\infty$ (from Figure 17). So the slope of the curve $y = f(x)$ approaches 1 as x becomes large (positive or negative). That is why we have drawn the graph of f in Figure 18 progressively straighter as $x \to \pm\infty$. ■

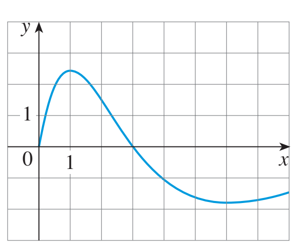
EXERCISES 3.2

1–2 Use the given graph to estimate the value of each derivative. Then sketch the graph of f'.

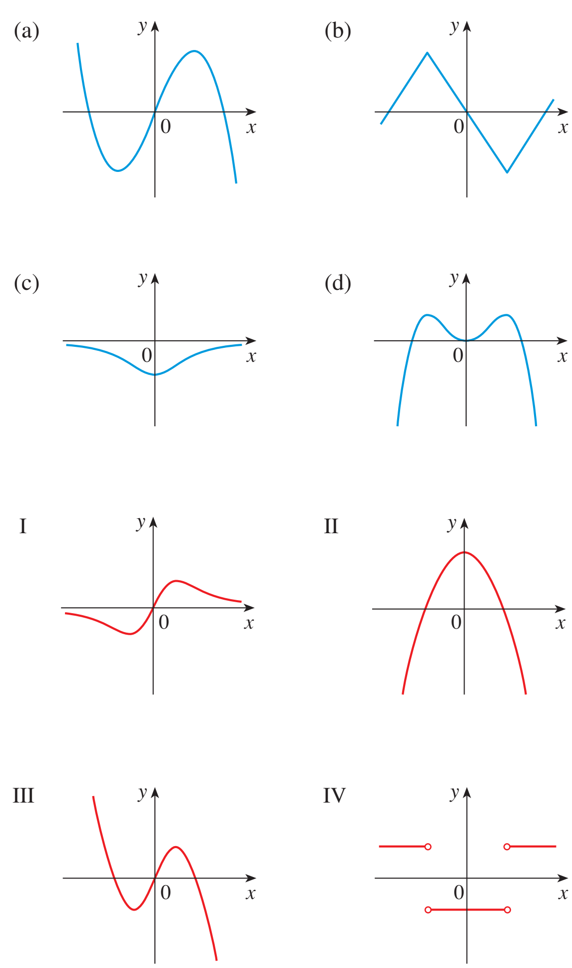
1. (a) $f'(-3)$ (b) $f'(-2)$
(c) $f'(-1)$ (d) $f'(0)$
(e) $f'(1)$ (f) $f'(2)$
(g) $f'(3)$



2. (a) $f'(0)$ (b) $f'(1)$
(c) $f'(2)$ (d) $f'(3)$
(e) $f'(4)$ (f) $f'(5)$
(g) $f'(6)$ (h) $f'(7)$

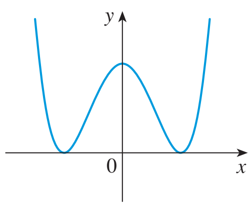


3. Match the graph of each function in (a)–(d) with the graph of its derivative in I–IV. Give reasons for your choices.

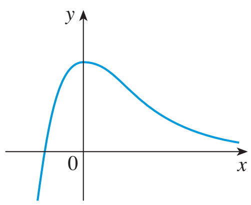


4–11 Trace or copy the graph of the given function f. (Assume that the axes have equal scales.) Then use the method of Example 1 to sketch the graph of f' below it.

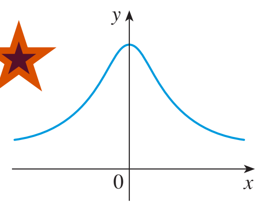
4.



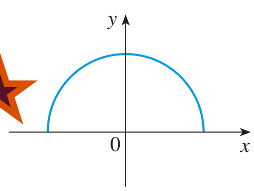
5.



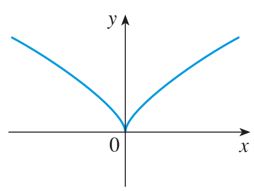
6.



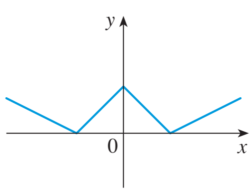
7.



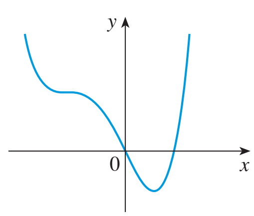
8.



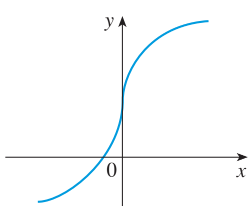
9.



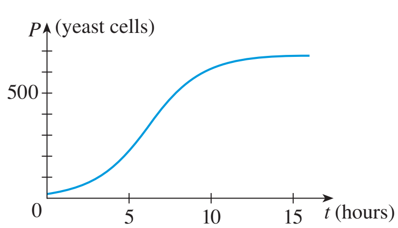
10.



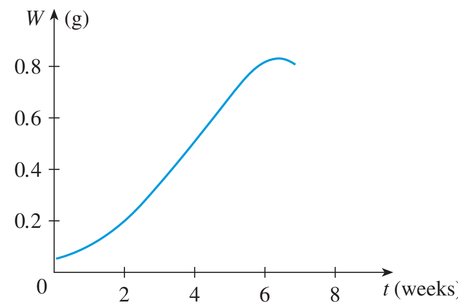
11.



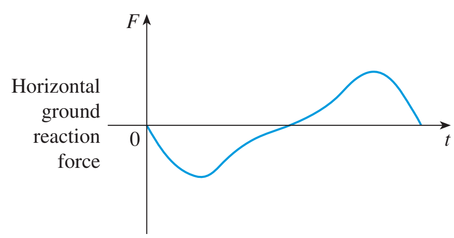
12. Yeast population Shown is the graph of the population function $P(t)$ for yeast cells in a laboratory culture. Use the method of Example 1 to graph the derivative $P'(t)$. What does the graph of P' tell us about the yeast population?



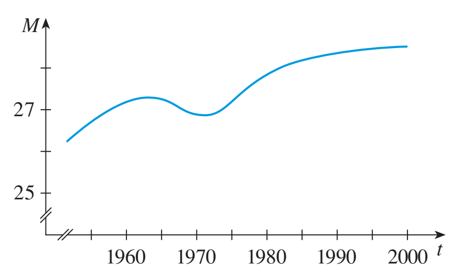
13. Tadpole weights The graph shows the average body weight W as a function of time for tadpoles raised in a density of 80 tadpoles/L.
(a) What is the meaning of the derivative $W'(t)$?
(b) Sketch the graph of $W'(t)$.



14. Ground reaction force in walking The graph shows the horizontal force $F(t)$ exerted by the ground on a person who is walking.
(a) What is the meaning of the derivative $F'(t)$?
(b) Sketch the graph of $F'(t)$.



15. Marriage age The graph shows how the average age of first marriage of Japanese men varied in the last half of the 20th century. Sketch the graph of the derivative function $M'(t)$. During which years was the derivative negative?



16–18 Make a careful sketch of the graph of f and below it sketch the graph of f' in the same manner as in Exercises 4–11. Can you guess a formula for $f'(x)$ from its graph?

16. $f(x) = \sin x$

17. $f(x) = e^x$

18. $f(x) = \ln x$

19. Let $f(x) = x^2$.
(a) Estimate the values of $f'(0)$, $f'(\frac{1}{2})$, $f'(1)$, and $f'(2)$ by using a graphing device to zoom in on the graph of f.
(b) Use symmetry to deduce the values of $f'(-\frac{1}{2})$, $f'(-1)$, and $f'(-2)$.
(c) Use the results from parts (a) and (b) to guess a formula for $f'(x)$.
(d) Use the definition of a derivative to prove that your guess in part (c) is correct.

20. Let $f(x) = x^3$.
(a) Estimate the values of $f'(0)$, $f'(\frac{1}{2})$, $f'(1)$, $f'(2)$, and $f'(3)$ by using a graphing device to zoom in on the graph of f.
(b) Use symmetry to deduce the values of $f'(-\frac{1}{2})$, $f'(-1)$, $f'(-2)$, and $f'(-3)$.
(c) Use the values from parts (a) and (b) to graph f'.
(d) Guess a formula for $f'(x)$.
(e) Use the definition of a derivative to prove that your guess in part (d) is correct.

21–31 Find the derivative of the function using the definition of a derivative. State the domain of the function and the domain of its derivative.

21. $f(x) = \frac{1}{2}x - \frac{1}{3}$

22. $f(x) = mx + b$

23. $f(t) = 5t - 9t^2$

24. $f(x) = 1.5x^2 - x + 3.7$

25. $f(x) = x^2 - 2x^3$

26. $f(x) = x + \sqrt{x}$

27. $g(x) = \sqrt{1 + 2x}$

28. $f(x) = \dfrac{x^2 - 1}{2x - 3}$

29. $G(t) = \dfrac{4t}{t + 1}$

30. $g(t) = \dfrac{1}{\sqrt{t}}$

31. $f(x) = x^4$

32–34
(a) Use the definition of the derivative to calculate f'.
(b) Check to see that your answer is reasonable by comparing the graphs of f and f'.

32. $f(x) = x + 1/x$

33. $f(x) = x^4 + 2x$

34. $f(t) = t^2 - \sqrt{t}$

35. Malarial parasites An experiment measured the number of malarial parasites $N(t)$ per microliter of blood, where t is measured in days. The results of the experiment are shown in the table.

t	N	t	N
1	228	5	372,331
2	2357	6	2,217,441
3	12,750	7	6,748,400
4	26,661		

(a) What is the meaning of $N'(t)$? What are its units?
(b) Construct a table of estimated values for $N'(t)$. [See Example 3.1.7 for $N'(3)$.]

36. Blood alcohol concentration Researchers measured the blood alcohol concentration $C(t)$ of eight adult male subjects after rapid consumption of 30 mL of ethanol (corresponding to two standard alcoholic drinks). The table shows the data they obtained by averaging the BAC (in mg/mL) of the eight men.

t (hours)	0.0	0.2	0.5	0.75	1.0	1.25
$C(t)$	0	0.25	0.41	0.40	0.33	0.29

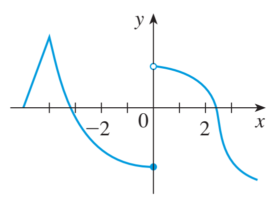
t (hours)	1.5	1.75	2.0	2.25	2.5	3.0
$C(t)$	0.24	0.22	0.18	0.15	0.12	0.07

(a) What is the meaning of $C'(t)$?
(b) Make a table of estimated values for $C'(t)$.

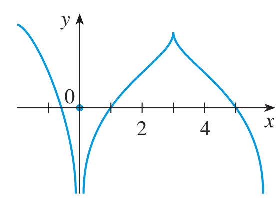
Source: Adapted from P. Wilkinson et al., "Pharmacokinetics of Ethanol after Oral Administration in the Fasting State," *Journal of Pharmacokinetics and Biopharmaceutics*, 5 (1977): 207–24.

37–40 The graph of f is given. State, with reasons, the numbers at which f is not differentiable.

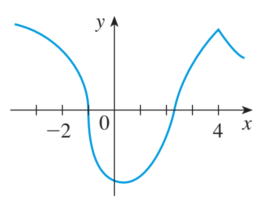
37.



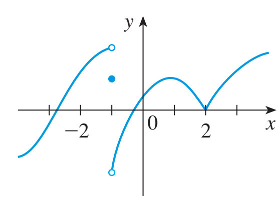
38.



39.



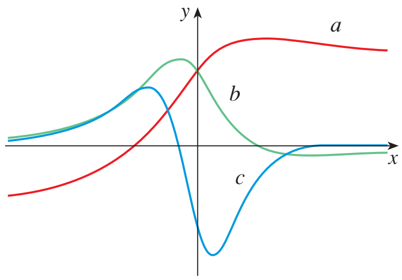
40.



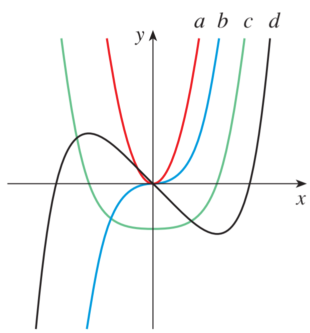
41. Graph the function $f(x) = x + \sqrt{|x|}$. Zoom in repeatedly, first toward the point $(-1, 0)$ and then toward the origin. What is different about the behavior of f in the vicinity of these two points? What do you conclude about the differentiability of f?

42. Zoom in toward the points $(1, 0)$, $(0, 1)$, and $(-1, 0)$ on the graph of the function $g(x) = (x^2 - 1)^{2/3}$. What do you notice? Account for what you see in terms of the differentiability of g.

43. The figure shows the graphs of f, f', and f''. Identify each curve, and explain your choices.



44. The figure shows graphs of f, f', f'', and f'''. Identify each curve, and explain your choices.



45–46 Use the definition of a derivative to find $f'(x)$ and $f''(x)$. Then graph f, f', and f'' on a common screen and check to see if your answers are reasonable.

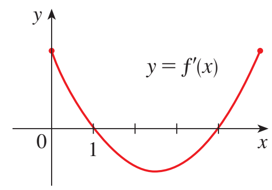
45. $f(x) = 3x^2 + 2x + 1$ **46.** $f(x) = x^3 - 3x$

47–48 The graph of the *derivative* f' of a function f is shown.

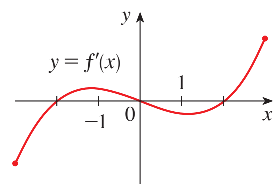
(a) On what intervals is f increasing?

(b) If it is known that $f(0) = 0$, sketch a possible graph of f.

47.



48.



49. Recall that a function f is called *even* if $f(-x) = f(x)$ for all x in its domain and *odd* if $f(-x) = -f(x)$ for all such x. Prove each of the following.

(a) The derivative of an even function is an odd function.

(b) The derivative of an odd function is an even function.

50. When you turn on a hot-water faucet, the temperature T of the water depends on how long the water has been running.

(a) Sketch a possible graph of T as a function of the time t that has elapsed since the faucet was turned on.

(b) Describe how the rate of change of T with respect to t varies as t increases.

(c) Sketch a graph of the derivative of T.

3.3 Basic Differentiation Formulas

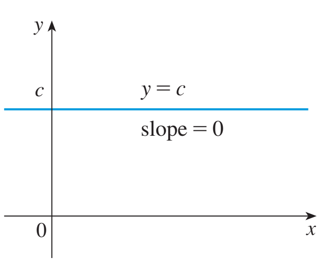


FIGURE 1
The graph of $f(x) = c$ is the line $y = c$, so $f'(x) = 0$.

If it were always necessary to compute derivatives directly from the definition, as we did in the preceding section, such computations would be tedious and the evaluation of some limits would require ingenuity. Fortunately, several rules have been developed for finding derivatives without having to use the definition directly. These formulas greatly simplify the task of differentiation.

In this section we learn how to differentiate constant functions, power functions, polynomials, exponential functions, and the sine and cosine functions.

Let's start with the simplest of all functions, the constant function $f(x) = c$. The graph of this function is the horizontal line $y = c$, which has slope 0, so we must have $f'(x) = 0$. (See Figure 1.) A formal proof, from the definition of a derivative, is also easy:

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} = \lim_{h \to 0} \frac{c - c}{h} = \lim_{h \to 0} 0 = 0$$

In Leibniz notation, we write this rule as follows.