Sec. 2.5

10. Sketch the graph of an example of a function f that satisfies all of the following conditions:

$$\lim_{x \to -\infty} f(x) = -2, \quad \lim_{x \to \infty} f(x) = 0, \quad \lim_{x \to -3^+} f(x) = \infty,$$

$$\lim_{x \to 3^-} f(x) = -\infty, \quad \lim_{x \to 3^+} f(x) = 2,$$

f is continuous from the right at 3

11–28 Find the limit.

11. $\lim_{x \to \infty} \dfrac{1 - x}{2 + 5x}$

12. $\lim_{t \to \infty} 3^{-2t}$

13. $\lim_{x \to 1} e^{x^3 - x}$

14. $\lim_{x \to 3} \dfrac{x^2 - 9}{x^2 + 2x - 3}$

15. $\lim_{x \to -3} \dfrac{x^2 - 9}{x^2 + 2x - 3}$

16. $\lim_{x \to 1^+} \dfrac{x^2 - 9}{x^2 + 2x - 3}$

17. $\lim_{h \to 0} \dfrac{(h - 1)^3 + 1}{h}$

18. $\lim_{t \to 2} \dfrac{t^2 - 4}{t^3 - 8}$

19. $\lim_{r \to 9} \dfrac{\sqrt{r}}{(r - 9)^4}$

20. $\lim_{v \to 4^-} \dfrac{4 - v}{|4 - v|}$

21. $\lim_{u \to 1} \dfrac{u^4 - 1}{u^3 + 5u^2 - 6u}$

22. $\lim_{x \to 3} \dfrac{\sqrt{x + 6} - x}{x^3 - 3x^2}$

23. $\lim_{x \to \pi^-} \ln(\sin x)$

24. $\lim_{x \to -\infty} \dfrac{1 - 2x^2 - x^4}{5 + x - 3x^4}$

25. $\lim_{x \to \infty} \dfrac{\sqrt{x^2 - 9}}{2x - 6}$

26. $\lim_{x \to \infty} e^{x - x^2}$

27. $\lim_{x \to \infty} \left(\sqrt{x^2 + 4x + 1} - x \right)$

28. $\lim_{x \to 1} \left(\dfrac{1}{x - 1} + \dfrac{1}{x^2 - 3x + 2} \right)$

29. The **Michaelis-Menten equation** for the rate v of the enzymatic reaction of the concentration [S] of a substrate S, in the case of the enzyme pepsin, is

$$v = \dfrac{0.50[S]}{3.0 \times 10^{-4} + [S]}$$

What is $\lim_{[S] \to \infty} v$? What is the meaning of the limit in this context?

30. Prove that $\lim_{x \to 0} x^2 \cos(1/x^2) = 0$.

31. Let

$$f(x) = \begin{cases} \sqrt{-x} & \text{if } x < 0 \\ 3 - x & \text{if } 0 \leq x < 3 \\ (x - 3)^2 & \text{if } x > 3 \end{cases}$$

(a) Evaluate each limit, if it exists.

   (i) $\lim_{x \to 0^-} f(x)$ (ii) $\lim_{x \to 0^-} f(x)$ (iii) $\lim_{x \to 0} f(x)$

   (iv) $\lim_{x \to 3^-} f(x)$ (v) $\lim_{x \to 3^+} f(x)$ (vi) $\lim_{x \to 3} f(x)$

(b)   Where is f discontinuous?

(c)   Sketch the graph of f.

32. Show that each function is continuous on its domain. State the domain.

(a) $g(x) = \dfrac{\sqrt{x^2 - 9}}{x^2 - 2}$ (b) $h(x) = xe^{\sin x}$

33–34 Use the Intermediate Value Theorem to show that there is a root of the equation in the given interval.

33. $2x^3 + x^2 + 2 = 0, \quad (-2, -1)$

34. $e^{-x^2} = x, \quad (0, 1)$

CASE STUDY 2a Hosts, Parasites, and Time-Travel

We are studying a model for the interaction between *Daphnia* and its parasite. Recall that there are two possible host genotypes (A and a) and two possible parasite genotypes (B and b). Parasites of type B can infect only hosts of type A, while parasites of type b can infect only hosts of type a. Here we will take equations that will be obtained in Case Studies 2b and 2d to explore the biological predictions that can be obtained from them.

In Case Study 2d we will derive the functions

(1a) $\qquad\qquad\qquad q(t) = \frac{1}{2} + M_q \cos(ct - \phi_q)$

(1b) $\qquad\qquad\qquad p(t) = \frac{1}{2} + M_p \cos(ct - \phi_p)$