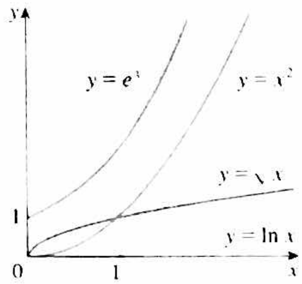

FIGURE 13

Looking at Figure 11 on page 107 we see that

$$\lim_{x \to \infty} e^x = \infty$$

It is also true that

$$\lim_{x \to \infty} x^2 = \infty \qquad \lim_{x \to \infty} \sqrt{x} = \infty \qquad \lim_{x \to \infty} \ln x = \infty$$

But as Figure 13 demonstrates, these four functions become large at different rates. In Chapter 4 we will see how to rank functions according to how quickly they grow.

EXAMPLE 10 | Find $\lim_{x \to \infty} (x^2 - x)$.

SOLUTION It would be **wrong** to write

$$\lim_{x \to \infty} (x^2 - x) = \lim_{x \to \infty} x^2 - \lim_{x \to \infty} x = \infty - \infty$$

The Limit Laws can't be applied to infinite limits because ∞ is not a number ($\infty - \infty$ can't be defined). However, we *can* write

$$\lim_{x \to \infty} (x^2 - x) = \lim_{x \to \infty} x(x - 1) = \infty$$

because both x and $x - 1$ become arbitrarily large and so their product does too. ■

EXAMPLE 11 | Find $\lim_{x \to \infty} \dfrac{x^2 + x}{3 - x}$.

SOLUTION As in Example 5, we divide the numerator and denominator by the highest power of x in the denominator, which is just x:

$$\lim_{x \to \infty} \frac{x^2 + x}{3 - x} = \lim_{x \to \infty} \frac{x + 1}{\dfrac{3}{x} - 1} = -\infty$$

because $x + 1 \to \infty$ and $3/x - 1 \to -1$ as $x \to \infty$. ■

EXERCISES 2.2

1. Explain in your own words the meaning of each of the following.
(a) $\lim_{x \to \infty} f(x) = 5$ (b) $\lim_{x \to -\infty} f(x) = 3$

2. (a) Can the graph of $y = f(x)$ intersect a horizontal asymptote? If so, how many times? Illustrate by sketching graphs.
(b) How many horizontal asymptotes can the graph of $y = f(x)$ have? Sketch graphs to illustrate the possibilities.

3. Guess the value of the limit

$$\lim_{x \to \infty} \frac{x^2}{2^x}$$

by evaluating the function $f(x) = x^2/2^x$ for $x = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 20, 50$, and 100. Then use a graph of f to support your guess.

4. (a) Use a graph of

$$f(x) = \left(1 - \frac{2}{x}\right)^x$$

to estimate the value of $\lim_{x \to \infty} f(x)$ correct to two decimal places.
(b) Use a table of values of $f(x)$ to estimate the limit to four decimal places.

5–28 Find the limit.

5. $\lim_{x \to \infty} \dfrac{1}{2x + 3}$

6. $\lim_{x \to \infty} \dfrac{3x + 5}{x - 4}$

7. $\lim_{x\to\infty} \dfrac{3x - 2}{2x + 1}$

8. $\lim_{x\to\infty} \dfrac{1 - x^2}{x^3 - x + 1}$

9. $\lim_{x\to\infty} \dfrac{1 - x - x^2}{2x^2 - 7}$

10. $\lim_{x\to\infty} \dfrac{4x^3 + 6x^2 - 2}{2x^3 - 4x + 5}$

11. $\lim_{t\to-\infty} 0.6^t$

12. $\lim_{r\to\infty} \dfrac{5}{10^r}$

13. $\lim_{t\to\infty} \dfrac{\sqrt{t} + t^2}{2t - t^2}$

14. $\lim_{t\to\infty} \dfrac{t - t\sqrt{t}}{2t^{3/2} + 3t - 5}$

15. $\lim_{x\to\infty} \dfrac{(2x^2 + 1)^2}{(x - 1)^2(x^2 + x)}$

16. $\lim_{x\to\infty} \dfrac{x^2}{\sqrt{x^4 + 1}}$

17. $\lim_{x\to\infty} \left(\sqrt{9x^2 + x} - 3x\right)$

18. $\lim_{x\to\infty} \left(\sqrt{x^2 + ax} - \sqrt{x^2 + bx}\right)$

19. $\lim_{x\to\infty} \dfrac{6}{3 + e^{-2x}}$

20. $\lim_{x\to\infty} \sqrt{x^2 + 1}$

21. $\lim_{x\to\infty} \dfrac{x^4 - 3x^2 + x}{x^3 - x + 2}$

22. $\lim_{x\to\infty} (e^{-x} + 2\cos 3x)$

23. $\lim_{x\to-\infty} (x^4 + x^5)$

24. $\lim_{x\to-\infty} \dfrac{1 + x^6}{x^4 + 1}$

25. $\lim_{t\to\infty} e^{-1/t^2}$

26. $\lim_{x\to\infty} \dfrac{e^{3x} - e^{-3x}}{e^{3x} + e^{-3x}}$

27. $\lim_{x\to\infty} \dfrac{1 - e^x}{1 + 2e^x}$

28. $\lim_{x\to-\infty} [\ln(x^2) - \ln(x^2 + 1)]$

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29. For the **Monod growth function** $R(N) = SN/(c + N)$, what is the significance of the constant c? [*Hint:* What is $R(c)$?]

30. The **Michaelis-Menten equation** models the rate v of an enzymatic reaction as a function of the concentration [S] of a substrate S. In the case of the enzyme chymotrypsin the equation is

$$v = \frac{0.14[S]}{0.015 + [S]}$$

(a) What is the horizontal asymptote of the graph of v? What is its significance?

(b) Use a graphing calculator or computer to graph v as a function of [S].

31. **Virulence and pathogen transmission** The number of new infections produced by an individual infected with a pathogen such as influenza depends on the mortality rate that the pathogen causes. This pathogen-induced mortality rate is referred to as the pathogen's *virulence*. [The photo shows victims at Camp Funston of the influenza epidemic of 1918.] Extremely high levels of virulence result in very little transmission because the infected individual dies before infecting other individuals. Under certain assumptions, the number of new infections N is related to virulence v by the function

$$N(v) = \frac{8v}{1 + 2v + v^2}$$

where v is the mortality rate (that is, virulence) and $v \geq 0$. Evaluate $\lim_{v\to\infty} N(v)$ and interpret your result.

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32. The **Bertalanffy growth function**

$$L(t) = L_\infty - (L_\infty - L_0)e^{-kt}$$

where k is a positive constant, models the length L of a fish as a function of t, the age of the fish. This model assumes that the fish has a well-defined length L_0 at birth ($t = 0$).

(a) Calculate $\lim_{t\to\infty} L(t)$. How do you interpret the answer?

(b) If $L_0 = 2$ cm and $L_\infty = 40$ cm, graph $L(t)$ for several values of k. What role does k play?

33. The **Pacific halibut fishery** has been modeled by the equation

$$B(t) = \frac{8 \times 10^7}{1 + 3e^{-0.71t}}$$

where $B(t)$ is the biomass (the total mass of the members of the population) in kilograms at time t. What is $\lim_{t\to\infty} B(t)$? What is the significance of this limit?

34. (a) A tank contains 5000 L of pure water. Brine that contains 30 g of salt per liter of water is pumped into the tank at a rate of 25 L/min. Show that the concentration of salt after t minutes (in grams per liter) is

$$C(t) = \frac{30t}{200 + t}$$

(b) What happens to the concentration as $t \to \infty$?