1. Suppose that 1,000 people are interested in attending ElvisLand. Once a person arrives at ElvisLand, his or her demand for rides is given by $x = \max\{ 6 - p, 0 \}$, where $p$ is the price per ride. There is a constant marginal cost of $3$ for providing a ride at ElvisLand. If ElvisLand charges a profit-maximizing two-part tariff, with one price for admission to ElvisLand and another price per ride for those who get in, how much should it charge per ride and how much for admission?

**Ans:**

The admission fee should be equal to the consumer surplus and the price per ride should be equal to the marginal cost which is $3.00$. The demand for rides at a price per ride of $3.00$ is $x=3$. The consumer surplus is the area under the demand curve for $0 \leq x \leq 3$ minus the square bounded by $(0,0)$, $(0,3)$, $(3,3)$ and $(3,0)$. The integral of $(6-x)$ between $0$ and $3$ is $13.5$. The area of the square is $9$, so the consumer surplus is $4.50$. Hence, the admission fee should be $4.50$ and the price per ride should be $3.00$. 

![Diagram](image)
2. Alice and Betsy are playing a game in which each can play either of two strategies, leave or stay. If both play the strategy leave, then each gets a payoff of $400. If both play the strategy stay, then each gets a payoff of $800. If one plays stay and the other plays leave, then the one who plays stay gets a payoff of $C$ and the one who plays leave gets a payoff of $D$. When is the outcome where both play leave a Nash equilibrium?

Ans:

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<thead>
<tr>
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<th>Leave</th>
<th>Stay</th>
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<tr>
<td>Betsy</td>
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<tr>
<td>Leave</td>
<td>($400, $400)</td>
<td>(D, C)</td>
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<tr>
<td>Stay</td>
<td>($C, D)</td>
<td>($800, $800)</td>
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When Betsy plays Leave, we want Alice to want to play leave, so $C < 400$. When Alice plays Leave, we want Betsy to want to play leave, so $C < 400$. (Leave, Leave) is a Nash equilibrium when $C < 400$. 
3. Ann and Bruce each own a pizza store in Frostbite Falls, Minnesota. Demand for pizza is given by \( Q = 200 - 10P \). Having the only two pizza stores in Frostbite Falls, they attempt to profitably split the market without violating the Sherman Antitrust Act. Each has the cost function \( C = 50 + 2Q \). If Ann and Bruce behave as duopolists, how much profit does each of them earn?

Ans:
The market price for pizza is given by \( p(Q) = 20 - (Q/10) \). If Bruce bakes \( q_B \) pizzas, and if Ann bakes \( q_A \) pizzas, total output will be \( q_A + q_B \) and price will be \( 20 - .1(q_A + q_B) \). For Ann, the total cost of producing \( q_A \) units of pizza is 50 + 2\( q_A \), so her profits are \( pq_A - C = (20 - .1q_A - .1q_B)q_A - (50 + 2q_A) = 20q_A - .1q_A^2 - .1q_A q_B - 50 - 2q_A = 18q_A - .1q_A^2 - .1q_A q_B - 50 \). Therefore, if Bruce is going to bake \( q_B \) units, then Ann will choose \( q_A \) to maximize \( 18q_A - .1q_A^2 - .1q_A q_B - 50 \). This expression is maximized when \( 18 - .2q_A - .1q_B = 0 \). Ann’s reaction function, \( R_A(q_B) \), tells us Ann’s best output if she knows that Bruce is going to bake \( q_B \). We solve to find \( R_A(q_B) = (18 - .1q_B)/2 = 90 - .5q_B \). Now we find Bruce’s reaction function. Since they both have the same market price and the same cost function, their reaction functions are mirror images, so Bruce’s reaction function is \( R_B(q_A) = 90 - .5q_A \). Now we solve these two equations in two unknowns: \( q_A = R_A(q_B) = 90 - .5q_B = 90 - .5(90 - .5q_A) = 90 - 45 + .25q_A \), so \( q_A = 60 \) and also \( q_B = 60 \).

Substituting into the profit function, \( 18q_A - .1q_A^2 - .1q_A q_B - 50 \), gives profit is equal to 310 each.
4. A mountain village owns a common pasture where villagers graze their goats. The cost to a goat owner of owning and caring for a goat is 4 groschen. The pasture gets overgrazed if too many goats share the pasture. The total revenue from all goats on the common pasture is \( f(g) = 48g - 2g^2 \), where \( g \) is the number of goats on the pasture. The town council notices that total profit from the pasture is not maximized if villagers are allowed to pasture goats for free. The council decides to allow a goat to use the common pasture only if its owner buys it a goat license. To maximize total profit (of villagers and council), how many groschen per goat should the council charge?

Ans:

Total revenue from all goats is \( f(g) = 48g - 2g^2 \). The cost to a villager is 4 groschen, so the total cost to the villagers is 4g groschen. Without a charge for grazing, profit is 48g - 2g^2 - 4g. Profit is maximized when there are 11 goats grazing, and profit is 242 groschen. Since the profit from each goat is 22 groschen and the cost of a goat is 4 groschen, villagers will want to have more goats grazing since the marginal revenue is greater than the marginal cost. Set the price for grazing a goat at \( p_c \) groschen, so the cost to the village for grazing is \( p_c g \) groschen. Then the profit of the villagers and the council is 48g - 2g^2 - 4g - \( p_c g \), so when g=11, profit is 48(11) - 2(11)^2 - 4(11) - 11p_c = 528 - 242 - 44 - 11p_c = 242 - 11p_c. The important thing is to not overgraze, so the price should be set to drive profit of all parties to zero. Then, 242 - 11p_c = 0, and \( p_c = 22 \) groschen.
5. Lucy’s utility function is $2X_L + G$ and Melvin’s utility function is $X_MG$, where $G$ is their expenditures on the public goods they share in their apartment and where $X_L$ and $X_M$ are their respective private consumption expenditures. The total amount they have to spend on private goods and public goods is $28,000. They agree on a Pareto optimal pattern of expenditures in which the amount that is spent on Lucy’s private consumption is $10,000. How much do they spent on public goods?

Ans:

We know that the public expenses plus Lucy’s private expenses plus Melvin’s private expenses cannot exceed the total amount they have to spend. This becomes $X_L + X_M + G = 28,000$. We also know that for a Pareto optimal pattern of expenditures, $|MRS_L| + |MRS_M| = MC(G)$. The $|MRS_L| = 1/2$. The $|MRS_M| = X_M/G$. The cost of the public goods is $C(G) = G$, so the marginal cost is 1. Then, $1/2 + X_M/G = 1$ and $G = 2X_M$.

Therefore, $X_L + X_M + G = 28,000$ becomes $X_L + 3X_M = 28,000$. Since Lucy’s private consumption is $10,000$, $3X_M = 18,000$, and $X_M = 6,000$, so $G = 12,000$. 