Problem Set 4  
(due in class March 9, 2012)

1. A competitive firm has the short-run cost function \( c(y) = y^3 - 2y^2 + 5y + 6 \). Write down equations for:
   a. The firm’s average variable cost function
   b. The firm’s marginal cost function
   c. At what level of output is average variable cost minimized?
   d. Graph the short-run supply function for this firm, being careful to label the key points on the graph with the numbers specifying the exact prices and quantities at these points.

Ans:
   a. The short run cost function is made up of fixed cost and variable cost. The fixed cost is 6 and the variable cost is \( y^3 - 2y^2 + 5y \). To calculate average variable cost, divide the variable cost by the output, \( (y^3 - 2y^2 + 5y)/y \) to get \( y^2 - 2y + 5 \).
   b. The marginal cost function is the first derivative of the cost function, \( 3y^2 - 4y + 5 \).
   c. To minimize average variable cost take the first derivative of the answer to part (a) and set it equal to zero and solve for \( y \). The first derivative is \( 2y - 2 = 0 \), so \( y = 1 \).
   d. The AVC curve is U-shaped with its bottom at \( y = 1, c = 4 \). The marginal cost curve is also U-shaped. It bottoms out at \( y = 2/3 \) and crosses the AVC curve from below at \( y = 1 \).
2. The Lost Mountains of northern Iowa are inhabited by the rare Marshallian deer. Patches of grass are far apart in this rugged land. If a deer finds a fresh patch of grass and spends $h$ minutes grazing it, it gets the square root of $h$ units of grass. The deer compete for grass. When there are $n$ deer, it takes a deer $n^2$ minutes to find a fresh patch. A deer can survive if it gets 1 unit of grass every 200 minutes.

a. Find the average cost in time of a unit of grass if a deer gets $y$ units of grass from each patch.
b. How much time will an efficient deer spend in each patch when there are $n$ deer? (Hint: Minimize average cost.)
c. Since there is free entry into the deer business, the equilibrium population is the maximum number of efficient deer who can survive. How many is this?

Ans:
See Extra Credit Answers on the Website
3. Long ago, a kindly prince noticed the misery of his subjects. His subjects all had the same preferences and the same low incomes. The demand function of each subject for bread was \( q = 26 - p \), where \( p \) is the price of bread and \( q \) is the number of loaves per week. The supply of bread per capita per week was given by the function \( q = .3p \). The king declared since his subjects did not even get a loaf of bread per day, he would help them by making it illegal to sell bread for more than 10 groschens per loaf. Unhappily, a bread shortage arose and people waited in long lines to get bread.

a. Draw a graph to show why. Put numerical labels on the important points on your graph.

b. If the citizens could earn 4 groschens per hour at work that was exactly as unpleasant as waiting in line, what would be the equilibrium waiting time for a loaf of bread?

Ans:

a. Equilibrium is when supply equals demand, when the price is 20 groschens and the supply is 6 loafs. When the prince fixes the price at 10 groschens, the supply drops to 3 loafs, but the demand increases to 16 loafs, so with demand exceeding supply, by more than 5 times, there are long lines.

b. The gain from waiting in line is 13 groschens because when the quantity is 3 the price the buyers are willing to pay is 23 groschens \( (p=26-q) \) and the price fixed by the prince is 10 groschens, so the gain to the buyers is 13 groschens. At 4 groschens per hour, it takes 3.25 hours to breakeven.
4. A baseball team’s attendance depends on the number of games it wins per season and on the price of its tickets. The demand function it faces is $Q = N(20 – p)$, where $Q$ is the number of tickets (in hundred thousands) sold per year, $p$ is the price per ticket, and $N$ is the fraction of its games that the team wins. The team can increase the number of games it wins by hiring better players. If the team spends $C$ million dollars on players, it will win $0.7 – 1/C$ of its games. Over the relevant range, the marginal cost of selling an extra ticket is zero.

a. Write an expression for the firm’s profits as a function of ticket price and expenditure on players.

b. Find the ticket price that maximizes revenue.

c. Find the profit-maximizing expenditure on players and the profit–maximizing fraction of games to win.

Ans:

a. Profit is Revenue minus Cost. Revenue is the number of tickets sold per year, $Q = N(20 – p)$, times the price per ticket, $p$. $N$ is the fraction of its games that the team wins, $0.7 – 1/C$ of its games, if the team spends $C$. Then, Profit = Revenue – Cost = $Qp – C = \frac{N(20 – p)p}{C} – C = 0.7 – \frac{1}{C}(20 – p)p – C$.

b. Take the derivative of the revenue function, $R = N(20 – p)p$ with respect to $p$ and set that equal to zero and solve for $p$. The derivative is $N(20-2p) = 0$, so $p = 10$.

c. Substitute $p = 10$ into the answer to part (a), take the derivative with respect to $C$, set that equal to zero and solve for $C$. Substituting gives Profit = $(0.7 – 1/C) (100) – C$. Taking the derivative and setting equal to zero gives $(100/C^2) – 1 = 0$, so $C = 10$. Then substitute $C = 10$ into $N = 0.7 – 1/C$ to get $0.60$. The team will win 60% of its games.