Problem Set 2
(due in class February 6, 2012)

1. Harvey’s net demands for goods 1 and 2 are (2, –3) and his endowment is (6, 5).
   a. What are his gross demands?
   b. Draw a diagram illustrating his budget line, his endowment, and his consumption. (Put good 1
      on the horizontal axis.)
   c. Draw a dotted line to show what his budget line would be if the price of good 1 doubled and
      the price of good 2 stayed the same.

   a. The definition of the endowment is:
   \[
   \text{Endowment} = \text{Gross Demands} - \text{Net Demands}.
   \]
   Stated another way, \[
   \text{Endowment} + \text{Net Demands} = \text{Gross Demands}.
   \]
   We know the Endowment (6,5), we know the Net Demands (2,-3), so the Gross Demands are
   (8,2).
   b. The endowment is always on the budget line and the slope of the budget line is \(-p_1/p_2\). His
      consumption is his gross demands, so consumption is (8,2).
   c. If the price of good 1 doubles, slope doubles and the budget line rotates around the endowment.
2. Peter has an endowment of 3 units of good x and 5 units of good y. He can buy and sell x at a price of $100, and y at a price of $200. He receives an income of $700 as alimony from a former spouse.
   a. Draw Peter’s budget line for x and y. Show his initial endowment of x and y on your diagram.
   b. Calculate the amount of x that he could afford if he bought only x and the amount of y he afford if he bought only y.
   c. Write an equation for Peter’s budget.

   a. The endowment is always on the budget line and the slope of the budget line is \(-p_1/p_2 = -1/2\).

   b. His total assets are $700 alimony, 3 units of good x worth $100 each for $300, and 5 units of good y worth $200 each for $1,000. His assets total $2,000.
      If he were to buy only x, then we divide $2,000 by the price of x, $100, to get x = 20.
      If he were to buy only y, then we divide $2,000 by the price of y, $200, to get y = 10.

   c. His budget equation will take the form of \(p_x x + p_y y = m + p_x \omega_x + p_y \omega_y\).
      Substituting \(p_x = 100, p_y = 200, m = 700, \omega_x = 3, \text{ and } \omega_y = 5\), we get
      \[100x + 200y = 700 + (100)(3) + (200)(5) = 700 + 300 + 1000 = 2000,\]
      so \[100x + 200y = 2000.\]
Marilyn is a journalist. She is considering two possible jobs. One job is as an editor for a magazine. The other job is writing freelance articles and selling them to whoever will buy them. If she works for the magazine, she must spend 10 hours a day at work and commuting. She will be paid $130 a day net of commuting costs and taxes if she takes this job. If she writes freelance articles, she can work at home as many hours a day as she pleases. She estimates that she would earn $10 an hour after taxes if she does this. Her utility function is $U = (R^3)C$, where $R$ is the number of hours a day she spends not working or commuting and $C$ is her earnings.

a. If Marilyn chooses to freelance, how many hours will she work?

b. Calculate her utility in each job and identify which job she will choose.

a. We know that the number of hours that Marilyn works is $24 - R$, and if she works freelance, she earns $10 per hour, so her earnings are $C = 10(24 - R)$. Then, her utility is $U = (R^3)C = (R^3)(10)(24 - R) = 240R^3 - 10R^4$. To maximize utility, take the derivative of utility with respect to $R$, set equal to zero and solve for $R$.

\[
dU/dR = 720R^2 - 40R^3 = 0,\]

so $R = 720/40 = 18$. $R$ is the number of hours of leisure, so $24 - R = 6$ is the number of hours she will work. She will work 6 hours per day.

b. In the freelance job, $U = (R^3)C$, and she spends 6 hours a day at work, so her leisure time is $R = 18$, and $R^3 = 5832$. She works 6 hours per day at $10 per hour, so her earnings are $60. Substituting into $U = (R^3)C = (5832)(60) = 349,920$.

c. In the magazine job, $U = (R^3)C$, and she must spend 10 hours a day at work, so her leisure time is $R = 14$, and $R^3 = 2744$. She works 10 hours per day at $13 per hour, so her earnings are $130. Substituting into $U = (R^3)C = (2744)(130) = 356,720$.

Since her utility is higher with the magazine, she should take the job with the magazine.