Problem Set 1  
(due in class January 30, 2012)

1. Felicity is studying economics and political science. She can read 30 pages of political science per hour but only 5 pages of economics per hour. This week she has a 50-page assignment in economics and a 150-page assignment in political science. Because of sorority rush, she cannot devote more than 10 hours to studying these subjects this week. She realizes she cannot complete all of her assignments but is determined to complete at least 30 pages of her economics reading. Draw a graph with pages of economics on the horizontal axis and pages of political science on the vertical axis. On this graph, show the possibilities that are consistent with the constraints that Felicity has imposed on herself. (She is allowed to read ahead in either subject.) Label key points on your graph with their numerical values.

The objective is to draw a graph with possibilities that are consistent with the constraints that Felicity has imposed on herself. The possibilities that consistent with the constraints are the elements of the budget set. She has a time budget that is limited to 10 hours. A budget constraint is in the form of the familiar equation $p_1x_1 + p_2x_2 = m$. For this problem, let’s use $p_s x_s + p_e x_e = t$, where $p_s$ is the time-cost (hours per page) for political science reading, where $p_e$ is the time-cost (hours per page) for economics reading, $x_s$ is the number of pages of political science that are read, and $x_e$ is the number of pages of economics that are read.

We know that Felicity can read 30 pages of political science per hour, so it takes her 1/30 of an hour for each page. We know that Felicity can read 5 pages of economics per hour, so it takes 1/5 of an hour for each page, and we know that she has only 10 hours available, $t$. Therefore, the budget equation is $(1/30)x_s + (1/5)x_e = 10$.

If we set $x_s = 0$, Felicity reads 300 pages of political science and if we set $x_e = 0$, Felicity reads 50 pages of economics. Here is a graph of the budget constraint. The budget set is below this line.

Felicity has another constraint. She is determined to complete at least 30 pages of her economics reading. Therefore, we add a vertical line at Economics = 30.

The shaded area is the budget set, the possibilities that are consistent with the constraints.
2. Blanche Carter has devised a system for rating the males in her economics class. She cares about their intelligence and their looks. She has ranked each male on a scale of 1 to 5 for intelligence and 1 to 3 for looks. She defines a preference relation, \( R \), as follows: \( xRy \) if boy \( x \) scores at least as high as boy \( y \) in either looks or intelligence. Give an example to show that Blanche’s method of determining preferences might not lead to transitive preferences.

To show that the preferences are not transitive, we need to show that if \( xRy \) and \( yRz \) then \( xRz \) is not true.

If boy \( x = (1,2) \) and boy \( y = (3,1) \), then \( xRy \) since \( x \) has better looks than \( y \), 2 versus 1. If boy \( z = (2,3) \) and boy \( y = (3,1) \), then \( yRz \) since \( y \) is more intelligent than \( z \), 3 versus 2. Now, when we examine \( x \) and \( z \) we find \( x \) is not more intelligent and not better looking than \( z \), so this preference relation does not hold. Therefore, the preference relation, \( R \), is not transitive.
3. Use separate graphs to sketch two indifference curves for people with each of the following utility functions:

(a) \( U(x, y) = x + 2y \).

Pick a utility level, say \( U(x, y) = k = x + 2y \). Then we can write \( y = -\frac{1}{2} x + \frac{1}{2} k \). This is a line with slope \(-\frac{1}{2}\) and \(y\)-intercept \(\frac{1}{2} k\). If we set \( k = 6 \), we get the solid line labeled \( U = 6 \), and if we set \( k = 12 \), we get the dashed line.

![Graph](image)

(b) \( U(x, y) = \min\{x, 2y\} \).

This is perfect compliments, the L-shape. Here we draw the ray from the origin, \( x = 2y \), or \( y = \frac{1}{2} x \). The L-shape will be along this line.

![Graph](image)

(c) \( U(x, y) = \max\{x, 2y\} \).

In this instance, the shape is like a 7 as opposed to an L, and we simply choose the larger. Again, we draw the ray from the origin, \( x = 2y \), or \( y = \frac{1}{2} x \). The 7-shape will be along this line.

![Graph](image)
4. Max has a utility function $U(x, y) = 2xy + 1$. The prices of $x$ and $y$ are both $1$ and Max has an income of $20$.

(a) How much of each good will he demand?
Assume that Max is a rational individual, so he will maximize his utility. From the above information, his budget constraint is $x + y = 20$. We can solve for $y = 20 - x$ and substitute into the utility function to get $U(x, y) = 2x(20 - x) + 1$. To maximize utility, we take the derivative with respect to $x$, set the derivative equal to zero, and solve for $x$. $dU/dx = 40 - 4x = 0$. Then, $x = 10$. Substituting back into the budget constraint, $x + y = 20$, we find that $y = 10$.

(b) A tax is placed on $x$ so that $x$ now costs Max $2$ while his income and the price of $y$ stay the same. How much of good $x$ does he now demand?
The new budget constraint is $2x + y = 20$. Using the same process as in part (a), we can solve for $y = 20 - 2x$ and substitute into the utility function to get $U(x, y) = 2x(20 - 2x) + 1$. To maximize utility, we take the derivative with respect to $x$, set the derivative equal to zero, and solve for $x$. $dU/dx = 40 - 8x = 0$. Then, $x = 5$. (Substituting back into the budget constraint, $2x + y = 20$, we find that $y = 10$.)

(c) Would Max be as well off as he was before the tax if when the tax was imposed, his income rose by an amount equal to $1$ times the answer to part (b)?
Now, Max’s income rises by $1$ times $5$ to $25$. The new budget constraint is $2x + y = 25$. Using the same process as in part (a), we can solve for $y = 25 - 2x$ and substitute into the utility function to get $U(x, y) = 2x(25 - 2x) + 1$. To maximize utility, we take the derivative with respect to $x$, set the derivative equal to zero, and solve for $x$. $dU/dx = 50 - 8x = 0$. Then, $x = 6.25$. Substituting back into the budget constraint, $2x + y = 25$, we find that $y = 12.5$.
To determine if Max is as well off as he was before the tax, part (a),
$U(x, y) = 2xy + 1 = 2(10)(10) + 1 = 201$.
After the tax, part (c), $U(x, y) = 2xy + 1 = 2(6.25)(12.5) + 1 = 157.25$
Clearly, Max is not as well off as he was before the tax if when the tax was imposed, his income rose by an amount equal to $1$ times the answer to part (b). Therefore, the answer is NO.
5. With some services, e.g., checking accounts, phone service, or pay TV, a consumer is offered a choice of two or more payment plans. One can either pay a high entry fee and get a low price per unit of service or pay a low entry fee and a high price per unit of service. Suppose you have an income of $100. There are two plans. Plan A has an entry fee of $20 with a price of $2 per unit. Plan B has an entry fee of $40 with a price of $1 per unit for using the service. Let \( x \) be expenditure on other goods and \( y \) be consumption of the service.

(a) Write down the budget equation that you would have after you paid the entry fee for each of the two plans.

For Plan A, after paying the entry fee, income is $80, \( p_x = 1 \) and \( p_y = 2 \).

Then the budget equation for Plan A is
\[
x + 2y = 80.
\]

For Plan B, after paying the entry fee, income is $60, \( p_x = 1 \) and \( p_y = 1 \).

Then the budget equation for Plan B is
\[
x + y = 60.
\]

(b) If your utility function is \( xy \), how much \( y \) would you choose in each case?

Assume that we are dealing with rational individuals who will maximize their utility.

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<thead>
<tr>
<th></th>
<th>For Plan A</th>
<th>For Plan B</th>
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<tbody>
<tr>
<td>Utility function</td>
<td>( U(x,y) = xy )</td>
<td>( U(x,y) = xy )</td>
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<tr>
<td>Demand Function</td>
<td>( x = 80 - 2y )</td>
<td>( x = 60 - y )</td>
</tr>
<tr>
<td>Utility function</td>
<td>( U(y) = (80 - 2y) y = 80y - 2y^2 )</td>
<td>( U(y) = (60 - 2y) y = 60y - y^2 )</td>
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<tr>
<td>Derivative (FOC)</td>
<td>( \frac{dU}{dy} = 80 - 4y = 0 )</td>
<td>( \frac{dU}{dy} = 60 - 2y = 0 )</td>
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<tr>
<td>( y = 20 )</td>
<td>( y = 30 )</td>
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<tr>
<td>( x = 40 )</td>
<td>( x = 30 )</td>
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(c) Which plan would you prefer? Explain.

Since we are dealing with a rational individual, the plan with the higher utility will be preferred.

The utility from Plan A is \( U(x,y) = xy = (20)(40) = 800 \).

The utility from Plan B is \( U(x,y) = xy = (30)(30) = 900 \).

Clearly, Plan B has higher utility and is preferred.
6. Suppose that the inverse demand function for wool is \( p = \frac{A}{q} \) for some constant \( A \). Suppose that \( \frac{1}{4} \) of the world’s wool is produced in Australia.

(a) If Australian wool production increases by 1% and the rest of the world holds its output constant, what will be the effect on the world price of wool?

The world demand for wool is \( q = \frac{A}{p} \).

The price elasticity of demand is \( \varepsilon = \frac{p}{q} \frac{dq}{dp} = \frac{p}{(A/p)}[-A/p^2] = -1 \).

So, wool has unit elastic demand. That means that if the price of wool falls by 1%, the quantity of wool demanded will increase by 1%. If Australian wool production increases by 1% and the rest of the world holds its output constant, then the total wool production in the world increases by 0.25% since only 1/4 of the world’s wool is produced in Australia. Therefore, as the quantity demanded increases by 0.25%, it must be true that the price falls by about 0.25%.

(b) How does the marginal revenue to Australia from an extra unit of wool relate to the price of wool? If the price falls by 0.25%, the new world price would be 99.75% of the old world price, or 0.9975 per unit. This new price would apply to the original 100% of production yielding a revenue loss of 0.0025 per unit or 0.25 for 100% of the units. Additional revenue of .9975 would come from the extra unit, so the marginal revenue would be 0.9975 – 0.25 = 0.7475, or about 75% of the price of wool.