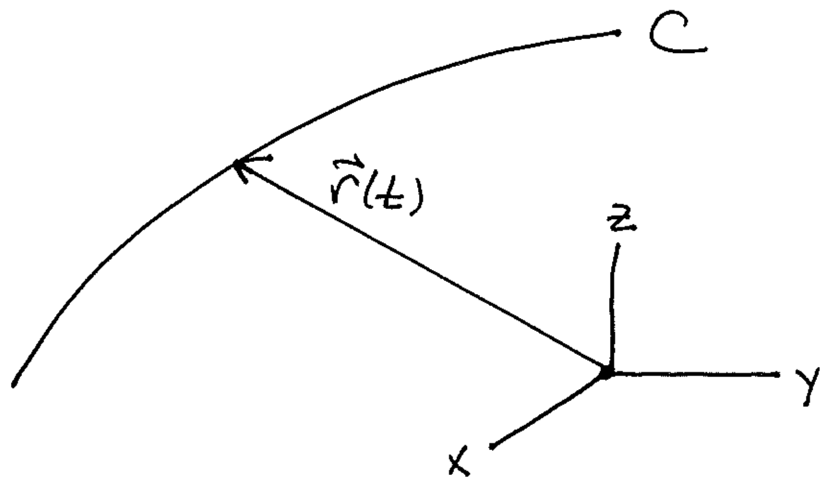
(13.1) VECTOR FUNCTIONS

DEFN.
LET $f(t), g(t), h(t)$ BE FUNCTIONS OF $t \in \mathbb{R}$, AND LET

$$\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$$
$$= f(t)\,\vec{i} + g(t)\,\vec{j} + h(t)\,\vec{k}$$

WE CALL $\vec{r}(t)$ A <u>VECTOR FUNCTION</u>.

GEOMETRICALLY, $\vec{r}(t)$ TRACES A CURVE C IN $\mathbb{R}^3$



SUCH A CURVE C IS SOMETIMES GIVEN IN TERMS OF ITS <u>PARAMETRIC EQUATIONS</u>.

$$\begin{cases} x = f(t) \\ y = g(t) \\ z = h(t) \end{cases}$$

Ex. $\vec{r}(t) = \langle \cos t, \sin t, t \rangle$

IN PARAMETRIC FORM: $\begin{cases} x = \cos t \\ y = \sin t \\ z = t \end{cases}$

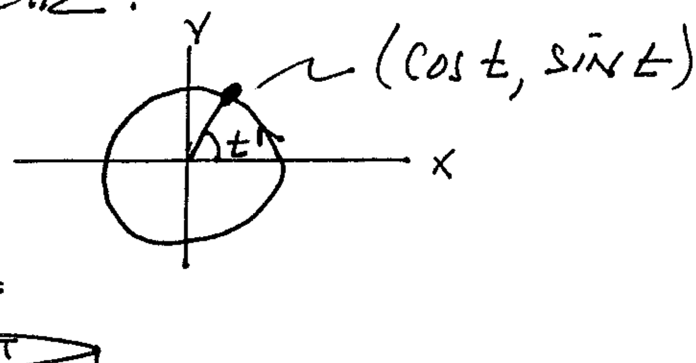
OBSERVE THAT FOR ALL $t \in \mathbb{R}$ WE HAVE

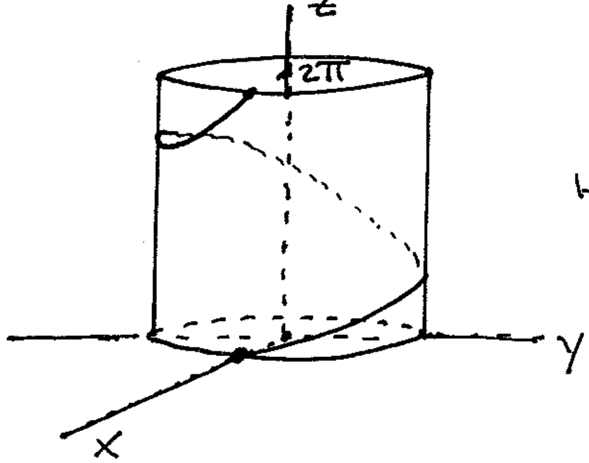
$$x^2 + y^2 = \cos^2 t + \sin^2 t = 1$$

$\therefore$ THIS CURVE LIES ON THE CYLINDER WITH EQUATION $x^2 + y^2 = 1$.

THE ORTHOGONAL PROJECTION OF $\vec{r}(t)$ INTO THE xy-PLANE IS THE STANDARD PARAMETRIZATION OF THE UNIT CIRCLE:

$$\begin{cases} x = \cos t \\ y = \sin t \end{cases}$$





HELIX

DEFN

$$\lim_{t \to a} \vec{r}(t) = \left\langle \lim_{t \to a} f(t), \lim_{t \to a} g(t), \lim_{t \to a} h(t) \right\rangle$$

THE limit ON THE LEFT is SAID TO EXIST PROVIDED EACH OF THE limits ON THE RIGHT EXIST.

EX.

$$\lim_{t \to 0} \left\langle \frac{e^t - 1}{t}, \frac{\sin t}{t}, \frac{1}{1+t} \right\rangle = \left\langle 1, 1, \frac{1}{2} \right\rangle$$

USING l'HOPITÂLS RULE.

DEFN

$\vec{r}(t)$ is SAID TO BE CONTINUOUS AT $t = a$ IFF

$$\lim_{t \to a} \vec{r}(t) = \vec{r}(a)$$

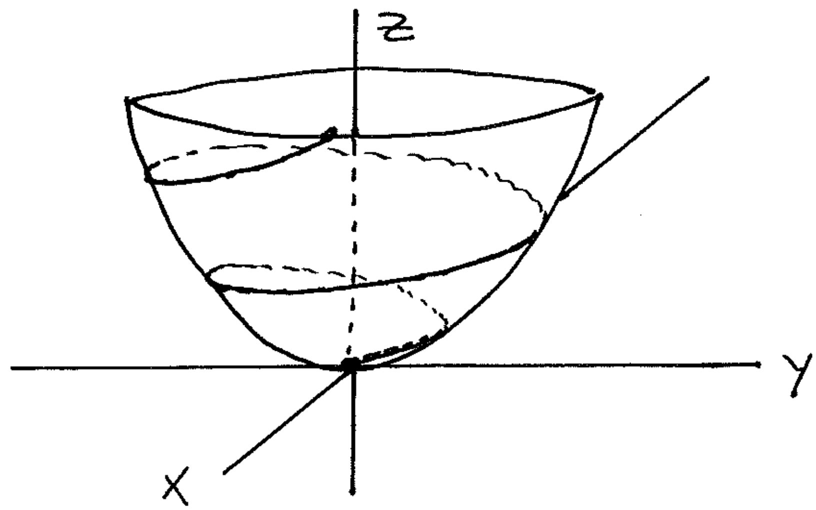
i.e. IFF $\lim_{t \to a} f(t) = f(a)$, $\lim_{t \to a} g(t) = g(a)$, AND $\lim_{t \to a} h(t) = h(a)$,

i.e. IFF EACH COMPONENT FUNCTION is CONTINUOUS AT $t = a$.

EX. $\vec{r}(t) = \langle t\cos t,\ t\sin t,\ t^2 \rangle$

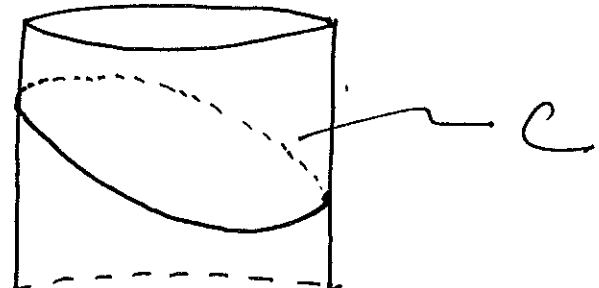
WE SEE THAT $\vec{r}(t)$ is CONTINUOUS FOR ALL t.

OBSERVE ALSO THAT $\vec{r}(t)$ SATISFIES $z = x^2 + y^2$, SO THE CURVE LIES ON AN ELLIPTIC PARABOLOID



EX. LET C BE THE INTERSECTION OF TWO SURFACES:

CYLINDER $x^2 + y^2 = 3$
PLANE $2x + 3y + z = 2$



C CAN BE PARAMETRIZED BY

$$\begin{cases} x = \sqrt{3}\cos t \\ y = \sqrt{3}\sin t \\ z = 2 - 2\sqrt{3}\cos t - 3\sqrt{3}\sin t \end{cases}$$

ALTERNATE PARAMETRIZATION

$$\begin{cases} x = \sqrt{3}\sin t \\ y = \sqrt{3}\cos t \\ z = 2 - 2\sqrt{3}\sin t - 3\sqrt{3}\cos t \end{cases}$$

WE SEE THAT THE CORRESPONDENCE FROM VECTOR FUNCTIONS TO CURVES IS MANY-TO-ONE, i.e. THERE ARE INFINITELY MANY PARAMETRIZATIONS OF THE VERY SAME CURVE.