7.3 FINDING EIGENVECTORS

DEFN:
LET λ BE AN EIGENVALUE OF $A \in M_n$.
THE EIGENSPACE BELONGING TO λ IS

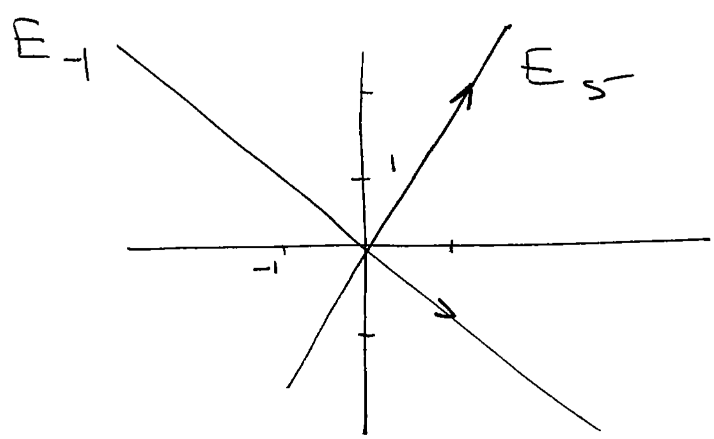
$$E_\lambda = \ker(A - \lambda I_n)$$
$$= \{ \vec{v} \in \mathbb{R}^n \mid A\vec{v} = \lambda \vec{v} \}.$$

THM: E_λ IS A SUBSPACE OF $\mathbb{R}^n$.

EX. $A = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$ $\lambda_1 = -1$, $\lambda_2 = 5$

$$E_{-1} = \ker\begin{pmatrix} 2 & 2 \\ 4 & 4 \end{pmatrix} = \ker\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} = \operatorname{span}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$E_5 = \ker\begin{pmatrix} -4 & 2 \\ 4 & -2 \end{pmatrix} = \ker\begin{pmatrix} 1 & -\frac{1}{2} \\ 0 & 0 \end{pmatrix} = \operatorname{span}\begin{pmatrix} 1 \\ 2 \end{pmatrix}.$$



OBSERVE THAT $\mathcal{B} = \left\{ \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\}$ is lin ind so

FORMS A BASIS OF $\mathbb{R}^2$ (CALLED AN <u>EIGEN-BASIS</u> FOR A)

THE MATRIX OF A WRT $\mathcal{B}$ is

$$B = \begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix}^{-1} \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 5 \end{pmatrix} \quad \text{(CHECK).}$$

<u>EX.</u> $A = \begin{pmatrix} 2 & 1 & 2 \\ 0 & 1 & 5 \\ 0 & 0 & 3 \end{pmatrix} \quad B = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

<u>EX.</u> $A = \begin{pmatrix} 3 & -2 & 5 \\ 1 & 0 & 7 \\ 0 & 0 & 2 \end{pmatrix} \quad E_1 = \text{SPAN}\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$

$E_2 = \text{SPAN}\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$

<u>DEFN.</u>
THE GEOMETRIC MULTIPLICITY OF λ is $\dim(E_\lambda)$.

<u>THM</u> $A \in M_n$ HAS AN EIGENBASIS IFF SUM OF ALGEBRAIC MULTIPLICITIES = SUM OF GEOMETRIC MULTIPLICITIES.

HW (7.3) 2-18 even