Math 11B
Midterm 2 Review Problems
Answers and hints to selected problems.

1. Evaluate the following indefinite integrals.

   d. $\int x^3 e^{x^2}\,dx = \frac{1}{2}e^{x^2}(x^2-1)+C$

   g. $\int \frac{10x+6}{(x+1)^2}\,dx = 10\ln|x+1| + \frac{4}{x+1} + C$

   h. $\int \frac{x^4+3x^3-2x^2+x+1}{x^2+2x+1}\,dx = \int\left(x^2 + x - 5 + \frac{10x+6}{(x+1)^2}\right)dx$, now see (g)

   i. $\int e^x \sin x\,dx = \frac{1}{2}e^x(\sin x - \cos x) + C$

2. Evaluate the following definite integrals.

   d. $\int_0^{\pi} e^x \sin x\,dx = \frac{e^{\pi}+1}{2}$

   e. $\int_0^{\pi/4} \tan x \sec^2 x\; e^{\tan x}\,dx$ Hint: let $u = \tan x$

   f. $\int_0^{\infty} \frac{1}{x^2+9}\,dx = \frac{\pi}{6}$

   g. $\int_0^{1} \frac{1}{x^{2/3}}\,dx = 3$

   h. $\int_1^{3} (x-2)^{-2/3}\,dx = 2\int_0^{1}\frac{1}{u^{2/3}}\,du$ upon doing the substitution $u = x-2$ and using symmetry

3. Determine the partial fraction decomposition of the following rational functions. (Do not integrate.)

   b. $\frac{x^2+x+1}{(x+1)(x^2+2)} = \frac{1/3}{x+1} + \frac{(2/3)x+(1/3)}{x^2+2}$

4. Determine the *form* of a partial fraction decomposition of the following rational functions. (Do not determine the constants.)

   b. $\frac{x^{10}+x^7+x^3+x+3}{(x^2+x+1)(2x^2+5)^2(x^2-1)^3} = \frac{x^{10}+x^7+x^3+x+3}{(x^2+x+1)(2x^2+5)^2(x-1)^3(x+1)^3}$

   $= \frac{Ax+B}{x^2+x+1} + \frac{Cx+D}{2x^2+5} + \frac{Ex+F}{(2x^2+5)^2} + \frac{G}{x-1} + \frac{H}{(x-1)^2} + \frac{I}{(x-1)^3} + \frac{J}{x+1} + \frac{K}{(x+1)^2} + \frac{L}{(x+1)^3}$

Note: I swapped the answers to #5 and #6 in class on Wednesday. The answers below are correct:

5. For which p does $\int_0^1 \frac{1}{x^p}\,dx$ converge? <u>Answer:</u> $p < 1$

6. For which p does $\int_1^\infty \frac{1}{x^p}\,dx$ converge? <u>Answer:</u> $p > 1$

7. Determine the convergence or divergence of the following improper integrals. (Do not evaluate the integrals.)
   a. $\int_1^\infty \frac{1}{\sqrt{x+\sqrt{x}}}\,dx$ (Hint: Show that $\frac{1}{\sqrt{2x}} \le \frac{1}{\sqrt{x+\sqrt{x}}}$ for all $x \ge 1$.) <u>Answer:</u> Diverges
   b. $\int_0^1 \frac{1}{\sqrt{x+\sqrt{x}}}\,dx$ (Hint: Show that $\frac{1}{\sqrt{x+\sqrt{x}}} < \frac{1}{\sqrt{x}}$ for all $0 < x \le 1$.) <u>Answer:</u> Converges

8. Solve the following initial value problem: $\begin{cases} \frac{dN}{dt} = te^t & t > 0 \\ N(0) = \frac{5}{2} \end{cases}$ <u>Solution</u>: $N(t) = (t-1)e^t + \frac{7}{2}$

9. Determine the average value of the following functions on the indicated intervals.
   a. $\ln x$ on $[1, e]$. <u>Answer</u>: $\frac{1}{e-1}$

10. Determine the area of the following plane regions.
    <u>Hint</u>: regions (b) and (c) have the same area (just flip the picture through the line $y = x$), but (b) is much easier to compute.
    b. The region in the 2nd quadrant bounded by $y = e^x$, $y = 0$, and $x = 0$.
    c. The region in the 4th quadrant bounded by $y = \ln x$, $y = 0$, and $x = 0$.