CHAPTER 7

TECHNIQUES OF INTEGRATION

7.1.1 SUBSTITUTION: INDEFINITE INTEGRALS

RECALL THE CHAIN RULE FOR DIFFERENTIATION

$$\frac{d}{dx} F(g(x)) = F'(g(x)) \cdot g'(x)$$

IF F IS AN ANTIDERIVATIVE OF f (i.e. $F' = f$) THIS READS

$$\frac{d}{dx} F(g(x)) = f(g(x)) \cdot g'(x)$$

WE THEREFORE HAVE THE INDEFINITE INTEGRAL FORMULA

$$\int f(g(x)) \cdot g'(x)\, dx = F(g(x)) + C$$

OFTEN IT IS DIFFICULT TO SEE HOW TO APPLY THIS FORMULA IN A SINGLE STEP.

WE APPLY A TECHNIQUE CALLED <u>u-SUBSTITUTION</u> WHICH HELPS US IDENTIFY g' AND $f \circ g$ WITHIN OUR INTEGRAND.

① LET $u = g(x)$

$$\therefore \frac{du}{dx} = g'(x) \quad \therefore du = g'(x)\,dx$$

② $$\int f(\underbrace{g(x)}_{u})\underbrace{g'(x)\,dx}_{du} = \int f(u)\,du$$

③ $$\int f(u)\,du = F(u) + C$$
$$= F(g(x)) + C$$

EX. RECALL FROM (6.3.5)

$$\int \sqrt{1 + \tfrac{9}{4}x}\,dx = \int \sqrt{u} \cdot \frac{4}{9}\,du$$

$u = 1 + \frac{9}{4}x$

$du = \frac{9}{4}\,dx$

$\frac{4}{9}\,du = dx$

$$= \frac{4}{9}\int u^{1/2}\,du$$
$$= \frac{4}{9} \cdot \frac{2}{3} \cdot u^{3/2} + C$$
$$= \frac{8}{27} u^{3/2} + C$$
$$= \boxed{\frac{8}{27}\left(1 + \frac{9}{4}x\right)^{3/2} + C}$$

Examples

$$\int 2x\sqrt{x^2+3}\,dx$$

$$\int 7x^3\sqrt{8+3x^4}\,dx$$

$$\int e^{3x+5}\,dx$$

$$\int x e^{-10x^2}\,dx$$

$$\int \frac{x^3-1}{x^4-4x}\,dx$$

$$\int \cos\left(\frac{3\pi}{2}x+\frac{\pi}{4}\right)dx$$

$$\int \cos(ax+b)\,dx$$

$$\int \tan x\cdot\sec^2 x\,dx$$

$$\int \cos x\cdot\sin x\,dx$$

Two ways: $u=\sin x$ vs. $u=\cos x$.