6.2.3 FTC (2nd Version)

The FTC (1st Version) says that
\[ \int_a^x f(u)\,du \]

is an antiderivative of \( f(x) \). Let \( F(x) \) stand for any other antiderivative. Then, as we saw in Sec. 5.8
\[ F(x) = \int_a^x f(u)\,du + C \]

for some \( C \in \mathbb{R} \). Hence
\[
F(b) - F(a) = \left[ \int_a^b f(u)\,du + C \right] - \left[ \int_a^a f(u)\,du + C \right] \\
= \int_a^b f(u)\,du + C - 0 + C
\]

Therefore
\[
\int_a^b f(u)\,du = F(b) - F(a). 
\]
We replace $u$ by $x$ to get

Thus (FTC \textsuperscript{no version})

Suppose $f(x)$ is continuous on $[a, b]$ and $F(x)$ is any antiderivative of $f(x)$ (i.e., $F'(x) = f(x)$). Then

\[
\int_{a}^{b} f(x) \, dx = F(b) - F(a)
\]

\textbf{Notation:} We write

\[
\left. F(x) \right|_{a}^{b} = F(b) - F(a)
\]

\textbf{or sometimes}

\[
\left. F(x) \right|_{x=a}^{x=b} = F(b) - F(a)
\]
\[ \text{Ex. } \int_2^4 (3-2x) \, dx = 3x - x^2 \bigg|_2^4 = (3 \cdot 4 - 4^2) - (3 \cdot 2 - 2^2) = -6 \]

\[ \text{Ex. } \int_0^1 (t^2 - \sqrt{t}) \, dt \]

\[ \text{Ex. } \int_1^8 \frac{1}{\sqrt[3]{x}} \, dx \]

\[ \text{Ex. } \int_0^2 (2x-1)(x+3) \, dx \]

\[ \text{Ex. } \int_0^{1/\sqrt{8}} \sec^2(2x) \, dx = \frac{1}{2} \tan(2x) \bigg|_0^{1/\sqrt{8}} = \frac{1}{2} \tan\left(\frac{1}{\sqrt{8}}\right) - \frac{1}{2} \tan(0) = \frac{1}{2} - 0 = \frac{1}{2} \]
Remember:

**Definite Integral**
\[ \int_{0}^{b} f(x) \, dx \text{ is a number} \]

**Indefinite Integral**
\[ \int f(x) \, dx \text{ is a family of functions} \]

**Ex.** \[ \int_{0}^{1} (x^{\frac{3}{5}} + x^{\frac{5}{3}}) \, dx \]

**Ex.** \[ \int (x^{\frac{3}{5}} + x^{\frac{5}{3}}) \, dx \]

---

**HW 3 6.2.4 (p. 371)**

2-10 even, 16-38 even, 44-64 even, 100-120 even.