

- lab 3: 1 absolutely last day (wed.)
- Pub: Posted

Problem: determine a k -coloring
 of a graph $\chi(G) \leq k \leq n = |V(G)|$
 $\uparrow$
 chromatic # of G .

new attributes:

- $\text{color}[x] : \{0, 1, 2, \dots, n\}$
 $\uparrow$
 assigned colors
 uncolored.

- $\text{ecs}[x] : \text{excluded color set of } x$

new functions

- $color()$

assign a coloring

return { colors used } = $\{1, 2, \dots, k\}$

- $getColor(x)$

returns $color[x]$

Basic Algorithm (for $color()$ function)

- 1.) start at some vertex
- 2.) while there are uncolored vertices
- 3.) find the best one, call it x
- 4.) $\leftarrow$ assign $color[x] =$ smallest color not in $ecs[x]$.
- 5.) $\leftarrow$ for all $y \in adj[x]$
- 6.) $\leftarrow$ add $color[x]$ to $ecs[y]$.

Suggestion:

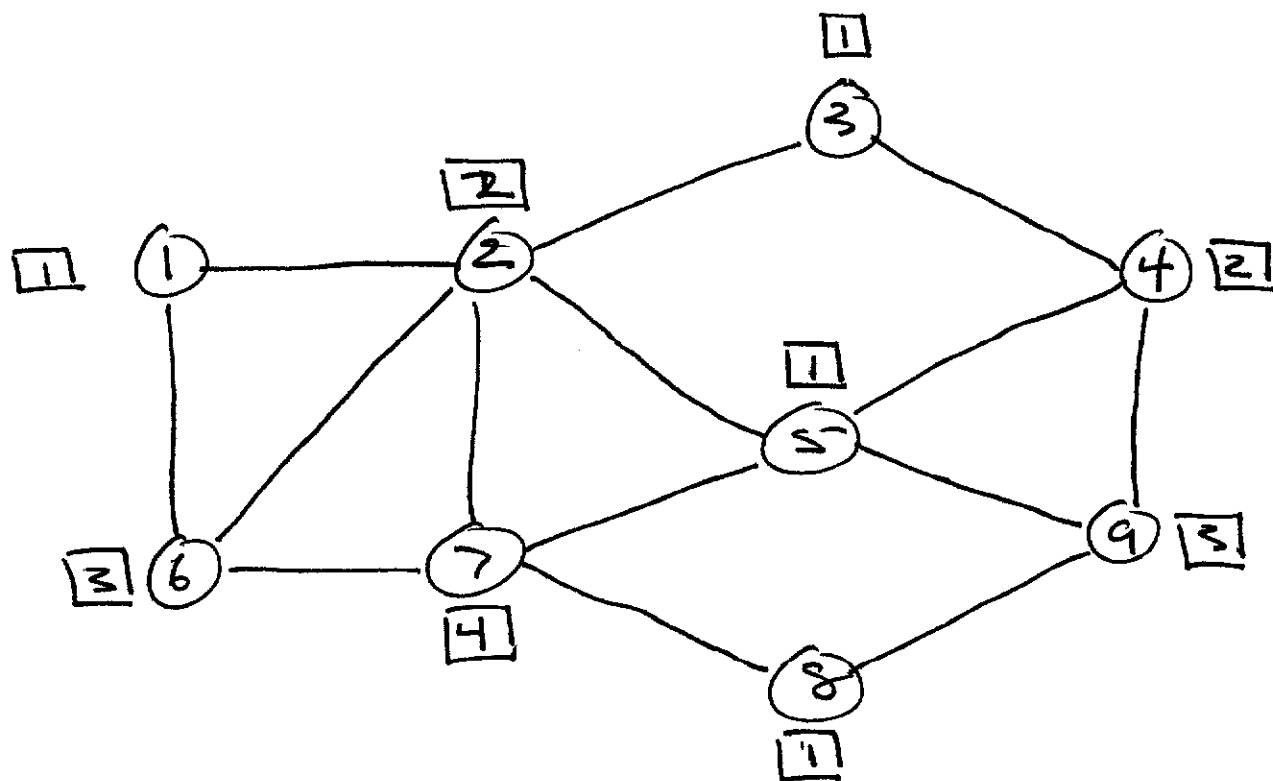
write helper function

- findBest(L) list of vertices
return the 'best' vertex
in list L.

Possible choices for 'best':

- highest degree
- lowest degree
- largest leaf
- smallest leaf
- closest to start (like BFS)
- mix strategies to break Tie.

Ex. start at 1, best = closest to start



used colours $\{1, 2, 3, 4\}$

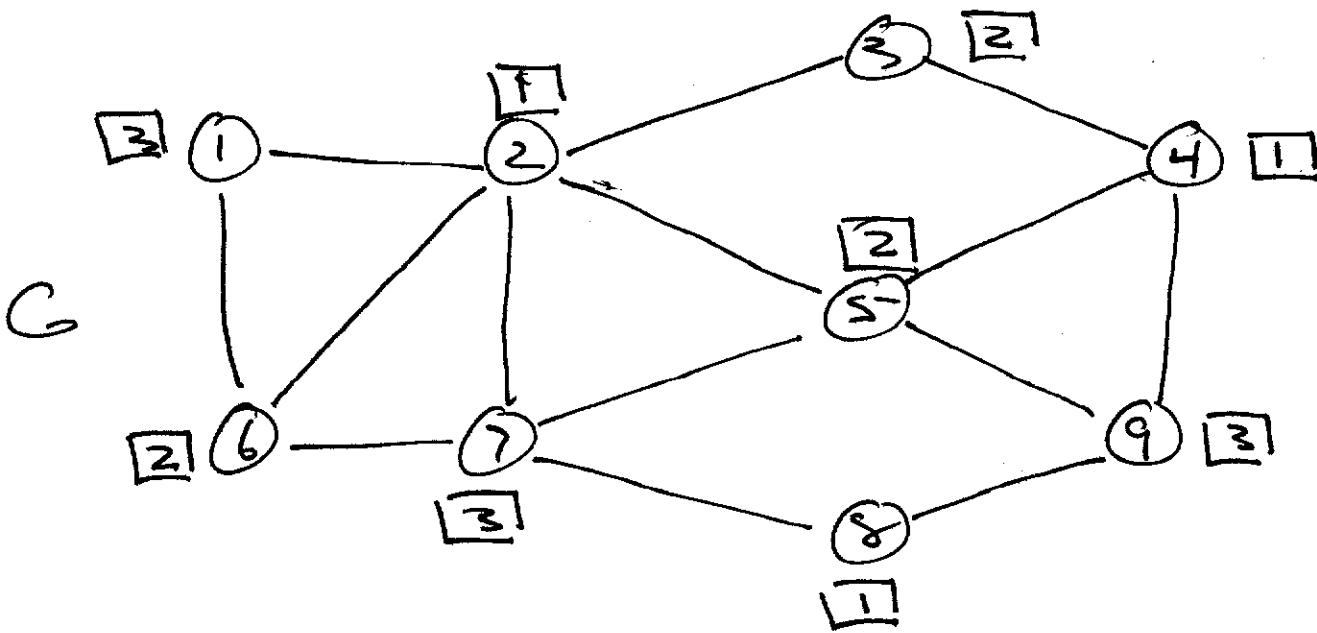
$k = 4$. now know $\chi(G) \leq 4$

Notice: this graph is in fact 3-colorable!!

Defn

The degree sequence of a graph is its set of vertex degrees arranged in descending order.

Ex. Same graph. best = highest deg.
 χ : 2 5 7 4 6 9 1 3 8
 deg seq: 5 4 4 3 3 3 2 2 2



used colors = $\{1, 2, 3\}$

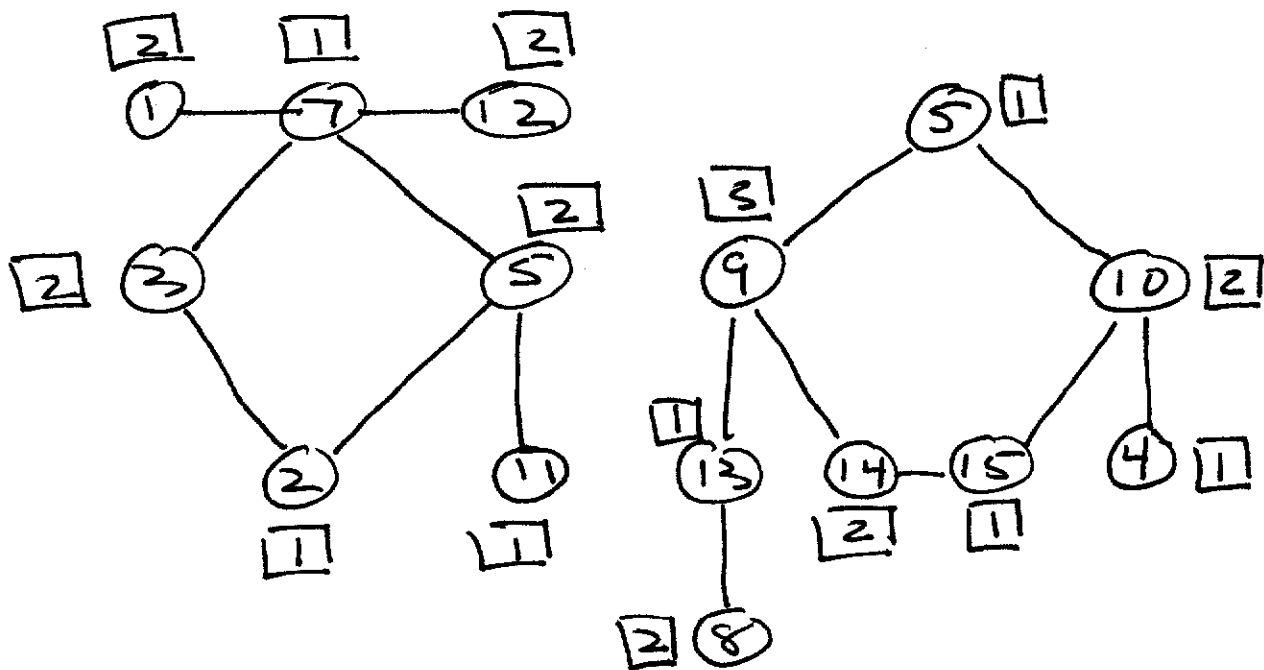
so $k=3$.

since G contains K_3 (in several ways), we have $\chi(G)=3$

Exercise:

- try lowest degree first
- try other strategies
- try other graphs

Ex. Graph in /Examples/pa6/m2



not connected. have 4-cycle & a 5-cycle, so $\chi(G) \geq 3$.

□

Theorem:

If H is a subgraph of G , then

$$\chi(H) \leq \chi(G).$$

strategy: start at highest deg.

best = smallest |ccs|

tiebreak: highest deg in adj[x]

deg. seq:

deg	4	3	3	3	2	2	2	2	2	2	1	1	1	1	1
x	7	6	9	10	2	3	5	13	14	15	1	4	8	11	12

Result: used colors = $\{1, 2, 3\}$

$$\chi = 3 \Rightarrow \chi(G) \leq 3$$

$$\therefore \chi(G) = 3$$