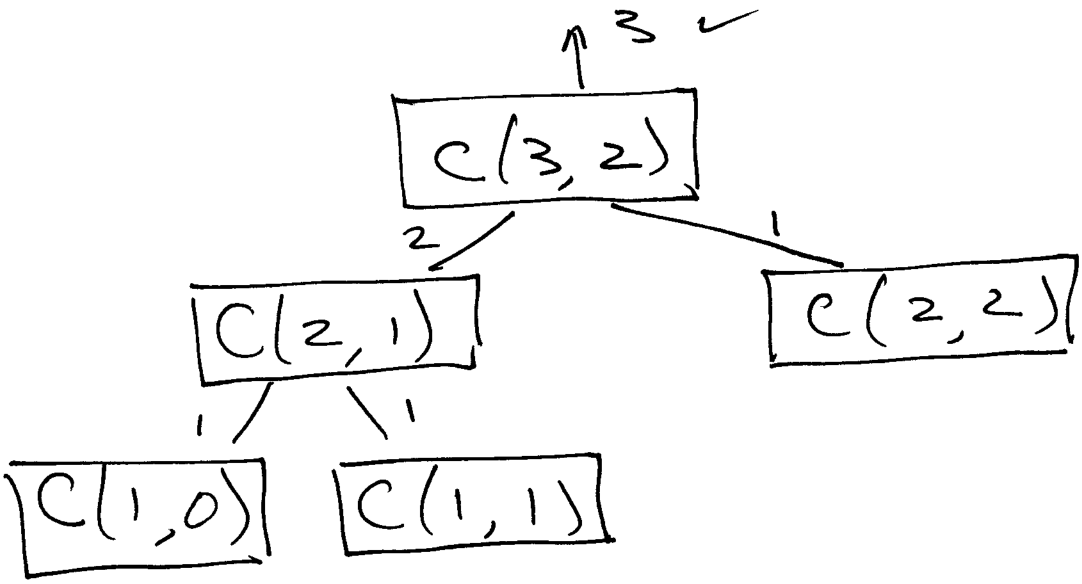
CSE 30 4-1-21

- Replacement Lecture for 4-1-21

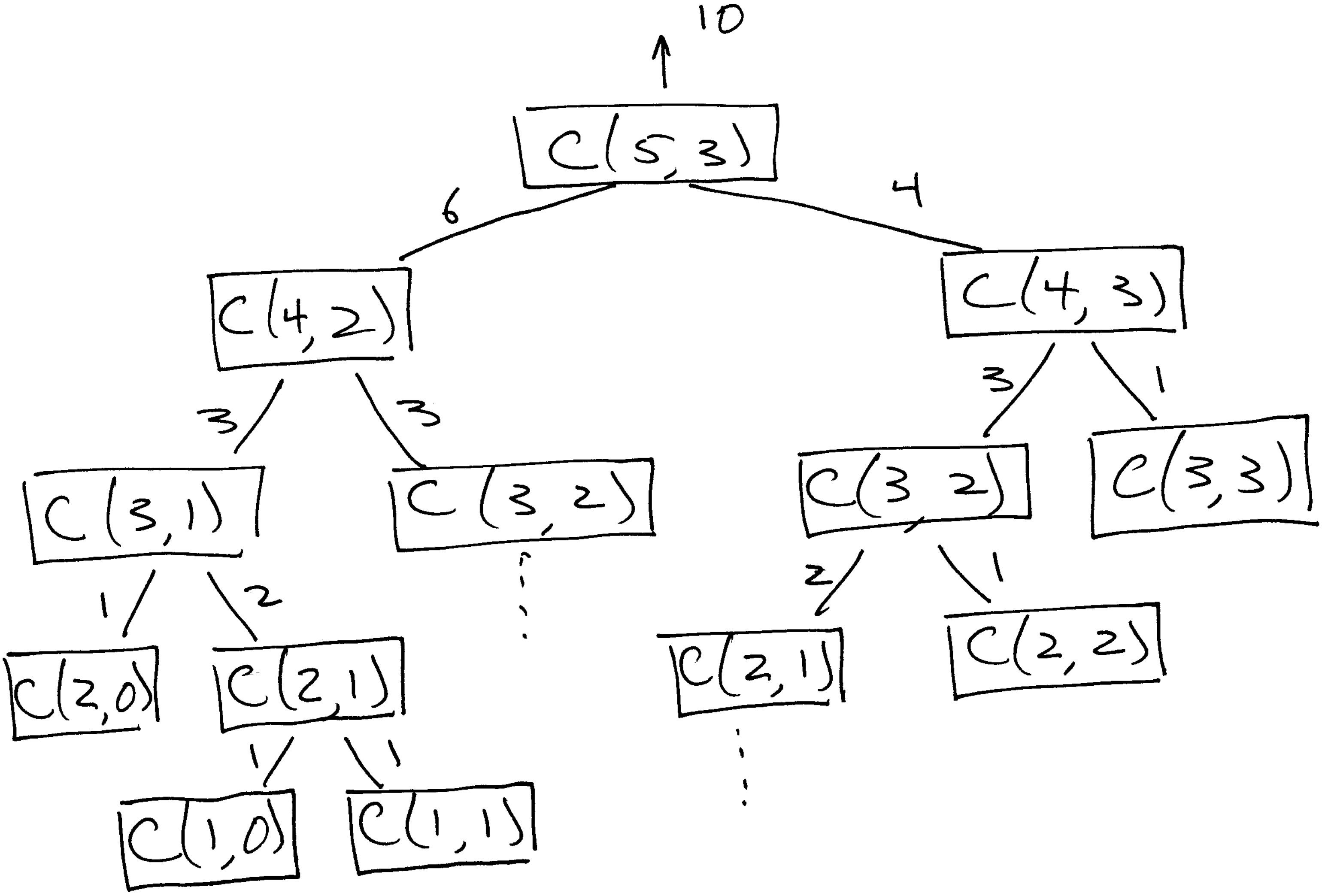
Binomial Coefficients

Box trace of $C(3,2) = \binom{3}{2} = 3$



Formula: $$\binom{n}{k} = \begin{cases} 1 & k=0,\ k=n \\ \binom{n-1}{k-1} + \binom{n-1}{k} & 0 < k < n \end{cases}$$

Box trace C(5, 3):



Memoization: keep a record of previously computed values, and use prev. comp. value instead of recursion, if you can.

Ex Binary Conversion: convert a number from base 10 to base 2.

$$\underbrace{x = 12}_{} = \overset{3}{(1}\ \overset{2}{1}\ \overset{1}{0}\ \overset{0}{0})_2 \quad \leftarrow \text{index}$$

let $x = 12 = (1100)_2$

$$= 1 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 0 \cdot 2^0$$

$$= 8 + 4 + 0 + 0$$

algorithm 1:

digit	
0	$x \% 2 = \boxed{0}$
	$x = x // 2 = 6$
1	$x \% 2 = \boxed{0}$
	$x = x // 2 = 3$
2	$x \% 2 = \boxed{1}$
	$x = x // 2 = 1$
3	$x \% 2 = \boxed{1}$
	$x = x // 2 = 0 \leftarrow$ stop.

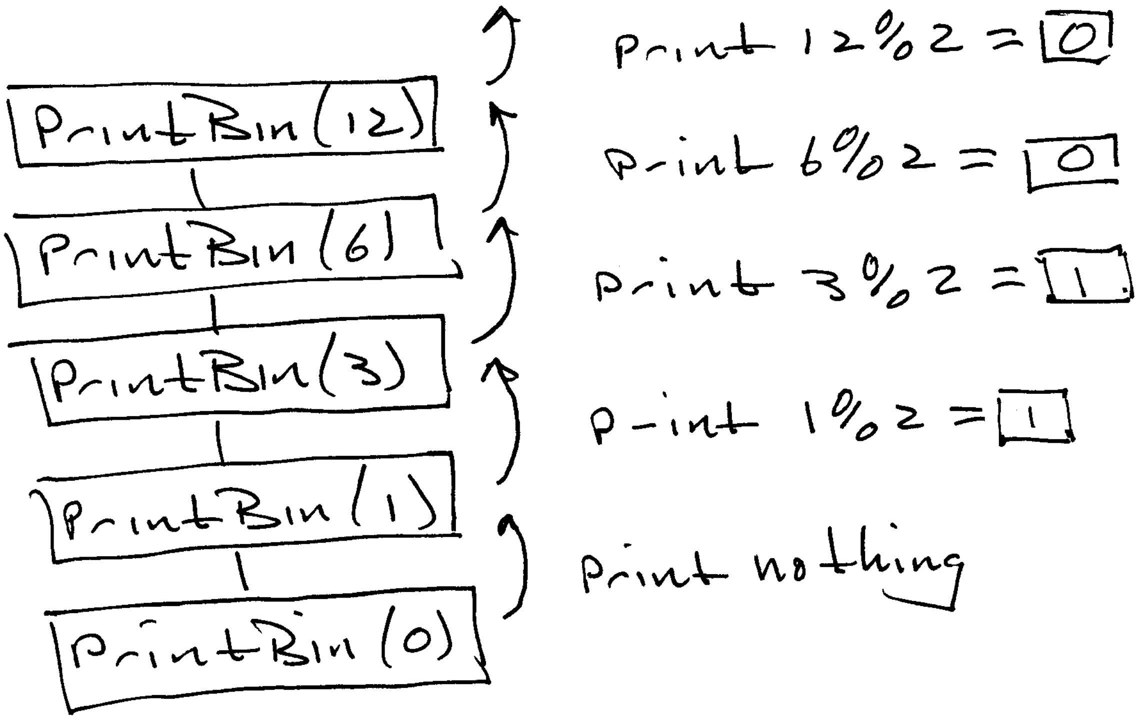
wrong order!!

can print in either order easily using recursion.

Version 3: <u>tail recursion</u>

- one recursive call
- is last operation.

Box trace of version 4 on $n = 12$.



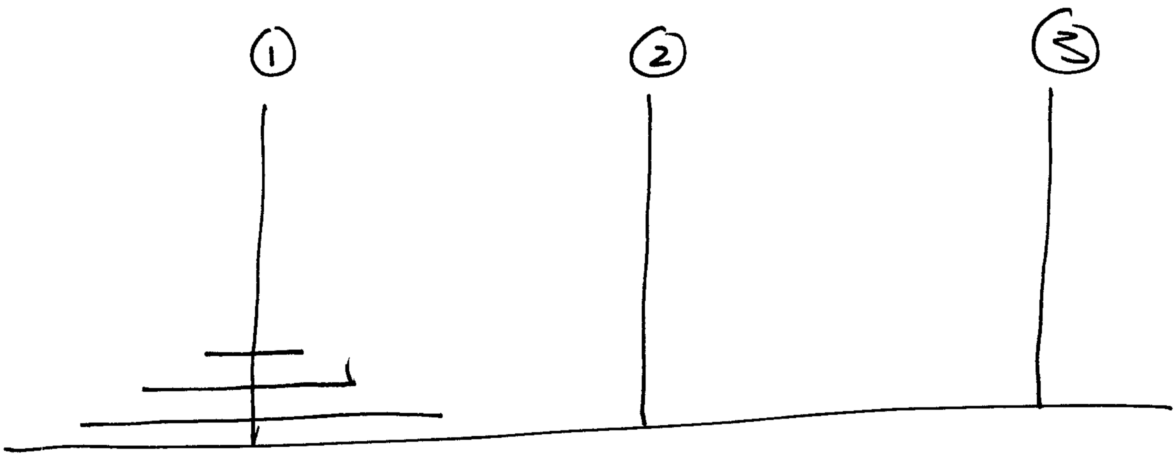
output: 1 1 0 0

Exercises : alter all 4 versions

so that

- print 0 when $x=0$.
- work with any base b.
- return a string instead of printing digits.

Ex. Tower of Hanoi Puzzle

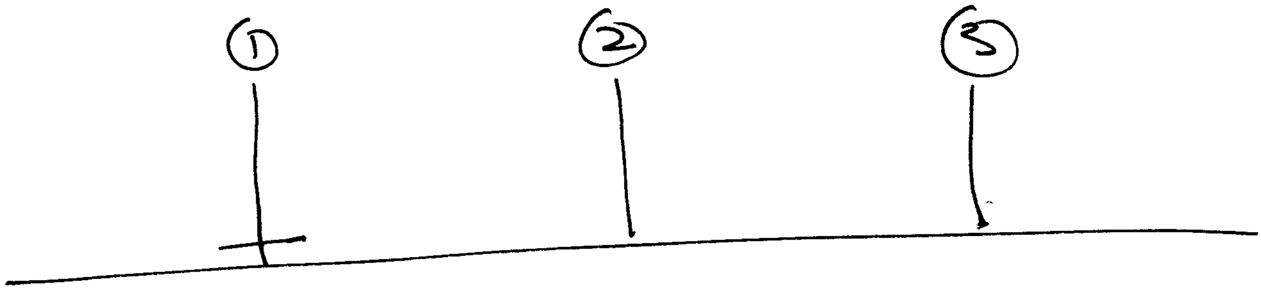


$n=3$: move all disks from 1 to 3

subject to : • one at a time

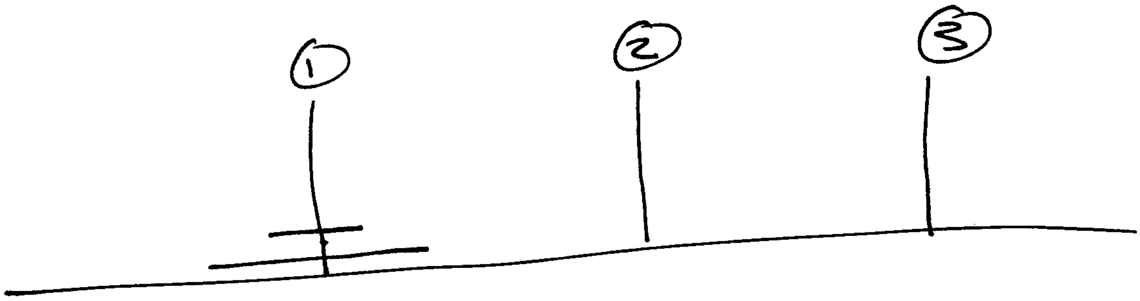
• never have larger above smaller.

Trivial version $n=1$.



$1 \to 3$

Case $n=2$: move from 1 to 3



$1 \to 2$

$1 \to 3$

$2 \to 3$

n=3 move disks from 1 to 3

move top 2 from 1 to 2

$1 \to 3$

move top 2 from 2 to 3

solution :

$1 \to 3$
$1 \to 2$
$3 \to 2$

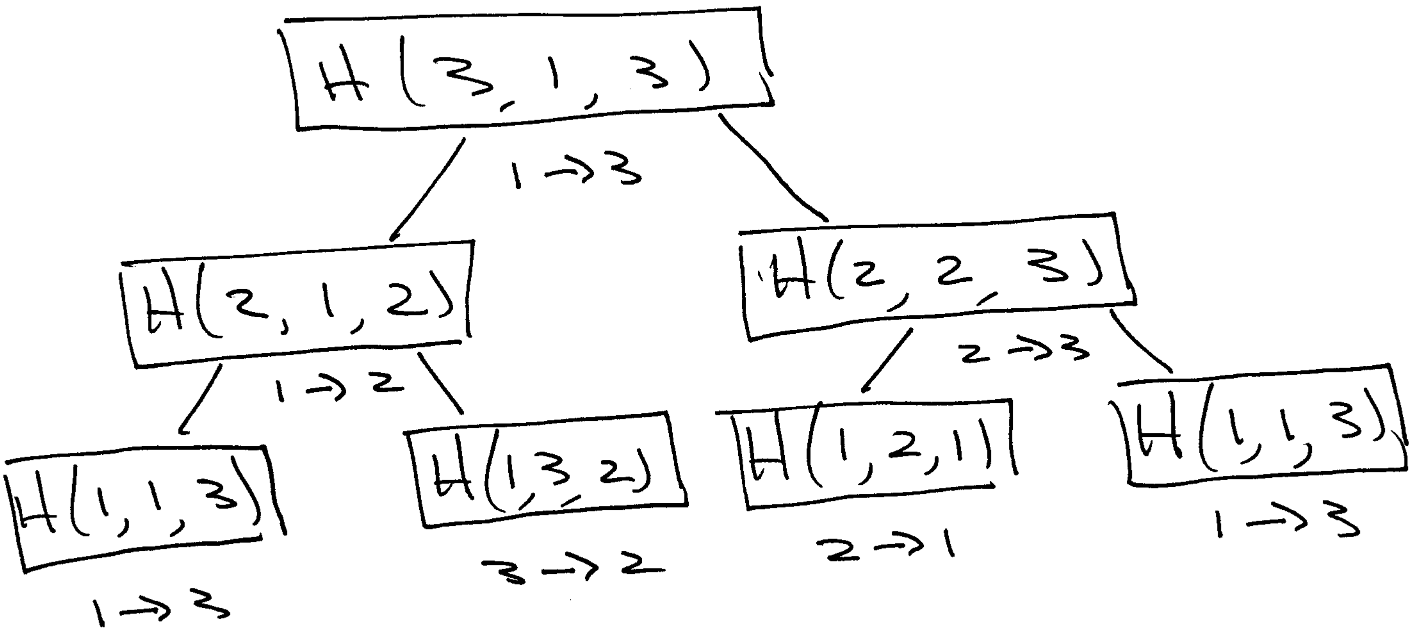
2 smallest
1 to 2

$1 \to 3$

$2 \to 1$
$2 \to 3$
$1 \to 3$

2 smallest
from 2 to 3

Box trace $H(3,1,3)$



how many instructions to move n disks?

$$T(n) = \#\text{ of instructions}$$

then

$$T(n) = T(n-1) + 1 + T(n-1)$$

$$T(n) = 2T(n-1) + 1$$

i.e.

$$T(n) = \begin{cases} 1 & n = 1 \\ 2T(n-1)+1 & n \geq 2 \end{cases}$$

solution: $\boxed{T(n) = 2^n - 1}$

$$\begin{aligned} RHS &= 2T(n-1)+1 \\ &= 2\cdot(2^{n-1}-1)+1 \\ &= 2\cdot 2^{n-1} - 2 + 1 \\ &= 2^n - 1 = T(n) = LHS. \end{aligned}$$

case $n = 10$.

$$T(10) = 2^{10} - 1 = 1024 - 1 = 1023$$