CSE 30   3-30-21

Recursion:

- Read de Alfaro Part I: Recursion

Ex factorial function

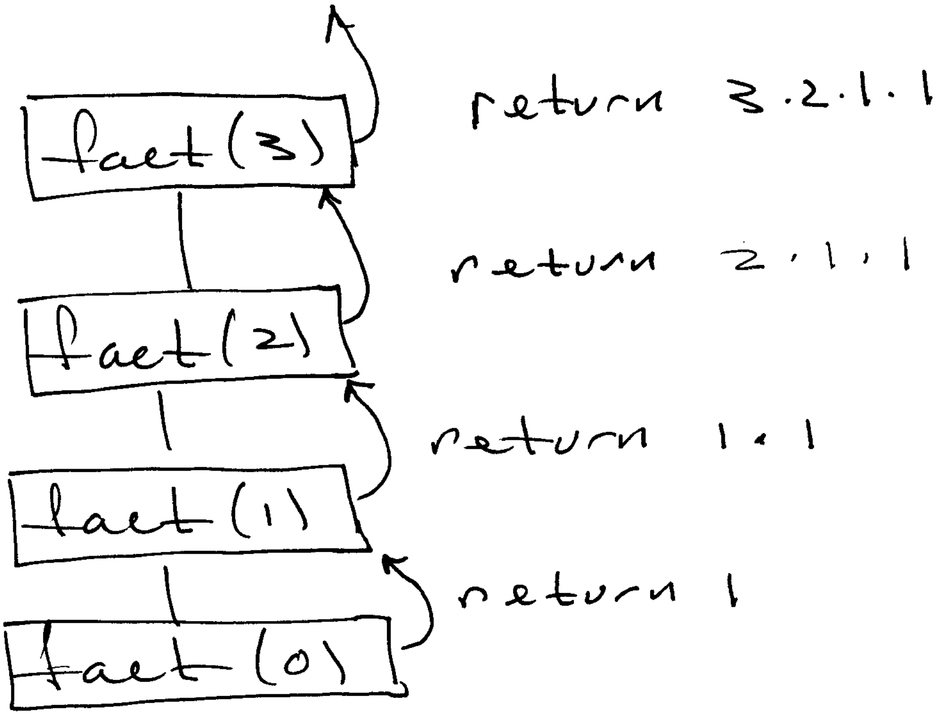
$$n! = \begin{cases} 1 & (\text{if } n=0) \\ \underline{n(n-1)(n-2)\cdots 3\cdot 2\cdot 1} & (\text{if } n \geq 1) \end{cases}$$

$$n! = \begin{cases} 1 & \text{if } n=0 \\ n\left[(n-1)!\right] & \text{if } n \geq 1 \end{cases}$$

- See Example Factorial.py

Box trace : a way of visualizing all recursive invocations of a function. each box represents one recursive invocation of the function.

Do factorial(3)



recursion 'bottoms out' here : base case, base instance.

Ex. Binomial Coefficients

Defn

$\binom{n}{k}$ = # of k-element subsets of an n-element set, for $0 \le k \le n$

"n-choose-k"

for example: $S = \{1, 2, 3\}$

0-element subsets of S: $\emptyset$

1- " " " " : $\{1\}, \{2\}, \{3\}$

2- " " " " : $\{1,2\}, \{1,3\}, \{2,3\}$

3- " " " " : $\{1, 2, 3\}$

so

$$\binom{3}{0} = 1, \quad \binom{3}{1} = 3, \quad \binom{3}{2} = 3, \quad \binom{3}{3} = 1$$

formula: $$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

__Theorem__ (Pascal's identity)

for $0 < k < n$:

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

Generates Pascal's triangle, with base cases: $\binom{n}{0} = \binom{n}{n} = 1$!

n														
0							1							
1						1		1						
2					1		2		1					
3				1		3		3		1				
4			1		4		6		4		1			
5		1		5		10		10		5		1		
6	1		6		15		20		15		6		1	
	⋮		⋮		⋮		⋮		⋮		⋮		⋱	

Proof of thm

Let S be an n-set and $0<k<n$.
Let x be any element in S.
Then the k-subsets of S fall into 2 categories:

(1) those that contain x:
Pick a $(k-1)$-subset of $S-\{x\}$, then add x. $\#$ ways $=\binom{n-1}{k-1}$

(2) those that do not contain x:
Pick a k-subset of $S-\{x\}$.
$\#$ ways $=\binom{n-1}{k}$

$\therefore$ the total $\#$ of k-subsets of S is $\binom{n-1}{k-1}+\binom{n-1}{k}$.

$$\therefore \binom{n}{k}=\binom{n-1}{k-1}+\binom{n-1}{k}.$$

$\blacksquare$