

CSE 201

Homework Assignment 6

- Moving on a checkerboard** (This is problem 15-6 on page 368 of the 2nd edition of CLRS.)
Suppose that you are given an $n \times n$ checkerboard and a single checker. You must move the checker from the bottom (1st) row of the board to the top (n^{th}) row of the board according to the following rule. At each step you may move the checker to one of three squares:
 - the square immediately above,
 - the square one up and one to the left (unless the checker is already in the leftmost column),
 - the square one up and one right (unless the checker is already in the rightmost column).

Each time you move from square x to square y , you receive $p(x, y)$ dollars. The values $p(x, y)$ are known for all pairs (x, y) for which a move from x to y is legal. Note that $p(x, y)$ may be negative for some (x, y) .

Give an algorithm that determines a set of moves starting at the bottom row, and ending at the top row, and which gathers as many dollars as possible. Your algorithm is free to pick any square along the bottom row as a starting point, and any square along the top row as a destination in order to maximize the amount of money collected. Determine the runtime of your algorithm.
- (This is Problem 15.2-1 on page 378 of CLRS.) Find an optimal parenthesization of a matrix-chain product whose sequence of dimensions is $(5, 10, 3, 12, 5, 50, 6)$. Show the complete tables $m[i, j]$ and $s[i, j]$ described in the handout on Dynamic Programming.
- (Read the **Rod-Cutting Problem** in section 15.1 pp. 360-369 of CLRS 3rd edition. This is problem 15.1-2 on p. 370.) Show, by means of a counterexample, that the following "greedy" strategy does not always determine an optimal way to cut rods. Define the *density* of a rod of length i to be p_i/i , that is, its value per inch. The greedy strategy for a rod of length n cuts off a first piece of length i , where $1 \leq i \leq n$, having maximum density. It then continues by applying the greedy strategy to the remaining piece of length $n - i$.