CSE 201
Homework Assignment 1

1. (Problem 3.1-1) Let $f(n)$ and $g(n)$ asymptotically positive functions. Prove that $f(n) + g(n) = \Theta(\max(f(n),\ g(n)))$.

2. Prove or disprove: If $f(n) = \Theta(g(n))$, then $f(n)^2 = \Theta(g(n)^2)$.

3. Prove or disprove: If $f(n) = \Theta(g(n))$, then $2^{f(n)} = \Theta(2^{g(n)})$.

4. Let $f(n)$ and $g(n)$ be asymptotically positive functions, and assume that $\lim_{n\to\infty} g(n) = \infty$. Prove that if $f(n) = \Theta(g(n))$, then $\ln\big(f(n)\big) = \Theta\big(\ln\big(g(n)\big)\big)$.

5. (Problem 3.2-8) Show that if $f(n)\ln f(n) = \Theta(n)$, then $f(n) = \Theta(n/\ln n)$. Hint: use the result of the preceding problem.

6. List the following 27 functions from lowest to highest asymptotic order. Indicate whether any two (or more) are of the same asymptotic order. Justify your answers.

$$2^{\lg(n)},\ \ (\lg n)^{\lg n},\ \ n^{1/\lg n},\ \ln(\ln(n)),\ \ 4^{\lg n},\ \ \sqrt{2}^{\lg n},\ \ n,\ \ \sqrt{2n},\ \ 1$$
$$n^2,\ \ n!,\ \ (3/2)^n,\ \ n^3,\ \ (\lg n)^2,\ \ 2^{2^n},\ \ n2^n,\ \ n^{\lg(\lg n)},\ \ \ln(n),\ \ e^n$$
$$2^{\sqrt{2\lg n}},\ \ 2^n,\ \ n\lg n,\ \ 2^{2^{n+1}},\ \ \lg(n!),\ \ \sqrt{\lg n},\ \ 2^{(\lg n)^2},\ \ (n+1)!$$

7. Determine the asymptotic order of the expression $\sum_{i=1}^{n} a^i$ where $a > 0$ is a constant, i.e. find a simple function $g(n)$ such that the expression is in the class $\Theta\big(g(n)\big)$. (Hint: consider the cases $a = 1$, $a > 1$, and $0 < a < 1$ separately.)

8. Use induction to prove that $\sum_{k=1}^{n} k^4 = \frac{n(n+1)(6n^3+9n^2+n-1)}{30}$ for all $n \geq 1$.