cse 201 2-27-20

Theorem

Greedy(M, w) returns an ind. set of max weight in S.

Proof.

Observe $A \in \mathcal{I}$ at each stage of execution, so set returned is independent. It's also maximal, by construction.

let B be any maximal ind. set in M, i.e. $B \subseteq S$ & $B \in \mathcal{I}$ & B has no extensions.

Then $|A| = |B| = \text{rank}(M) = r$.

must show: $w(A) \geq w(B)$, whence A is optimal. let

$$A = \{x_1, x_2, \ldots, x_r\}$$

and

$$B = \{y_1, y_2, \ldots, y_r\}$$

indexed by decreasing weights. In particular, A is ordered by selection in Greedy(M, w)

If $A = B$, then $w(A) = w(B)$, so done. Assume $A \neq B$.

let x_k be 1^{st} element of A not in B. i.e.

$$\begin{cases} x_i = y_i & (1 \le i \le k-1) \\ x_k \neq y_k & \end{cases}$$

let $A' = A - \{x_k\}$. Then $A' \subseteq A$ so $A' \in \mathcal{I}$, and

$$|A'| = |A| - 1 = |B| - 1 < |B|$$

By the exchange property, there exists $y \in B - A'$ s.t. $A' \cup \{y\} \in \mathcal{I}$.

<u>claim</u>: $w(x_k) \geq w(y)$.

<u>Proof</u>. observe $\{x_1, \ldots, x_{k-1}, y\} \subseteq A' \cup \{y\}$, so $\{x_1, \ldots, x_{k-1}, y\} \in \mathcal{I}$. If $w(x_k) < w(y)$, then Greedy(M, w) would have chosen y instead of x_k on k-th iteration $\times$.

$\square$

Let $A_1 = A' \cup \{y\} = (A - \{x_k\}) \cup \{y\}$.

Then $A_1 \in \mathrm{II}$ and by the above claim:

$$w(A) \geq w(A_1)$$

Also, A_1 has 1-more element in common with B than A does. If $A_1 = B$, done.

If $A_1 \neq B$, repeat with A_1 in place of A, to obtain $A_2 \in \mathrm{II}$ with $w(A_1) \geq w(A_2)$ and A_2 having yet another element in common with B. continuing

have seq. $\longrightarrow$ B :

$$w(\underset{\substack{\shortparallel \\ A_0}}{A}) \geq w(A_1) \geq w(A_2) \geq \cdots \geq w(B)$$

showing $w(A) \geq w(B)$. ■

Read: Unit time tasks

Sec 16.5.