Ex. TestPrime1.py

2.py

Defn

let a, b be positive integers. The gcd of a and b, denoted

$$\gcd(a,b)$$

is the largest integer d such that d divides a and d divides b.

Defn d divides n iff $n = kd$ for some integer k. notation $d \mid n$

'divides'

Euclid's Algorithm

<u>Ex</u> find $\gcd(42, 15)$

dividend quotient divisor remainder

$$42 = 2 \cdot 15 + 12$$

$$15 = 1 \cdot 12 + \boxed{3}$$ last non-zero remainder

$$12 = 4 \cdot \boxed{3} + 0$$ stop

last divisor is gcd.

<u>Ex.</u> Euclid.py

Ex. gcd(15, 42)

$$15 = 0 \cdot 42 + 15$$

$$42 = 2 \cdot 15 + 12$$

$$15 = 1 \cdot 12 + 3$$

$$12 = 4 \cdot \boxed{3} + 0$$

$$\downarrow$$

gcd

Collatz sequence

$$C(n) = \begin{cases} \frac{n}{2} & \text{if } n \text{ even} \\ 3n+1 & \text{if } n \text{ odd} \end{cases}$$

start at n:

$$x_0, \underbrace{C(x_0)}_{= x_1}, \underbrace{C(x_1)}_{= x_2}, \underbrace{C(x_2)}_{= x_3}, \ldots\ldots$$

start at 1:

$$1, 4, 2, 1, 4, 2, 1, \ldots\ldots$$

infinite cyclic.

<u>Rmk!</u> no one has found an x_0 s.t. $x_0, x_1, x_2, \ldots\ldots, \underset{\uparrow}{1}$

does not happen ... eventually

i.e. there is no known x_0 for which $x_0, x_1, x_2 \ldots \longrightarrow \infty$.

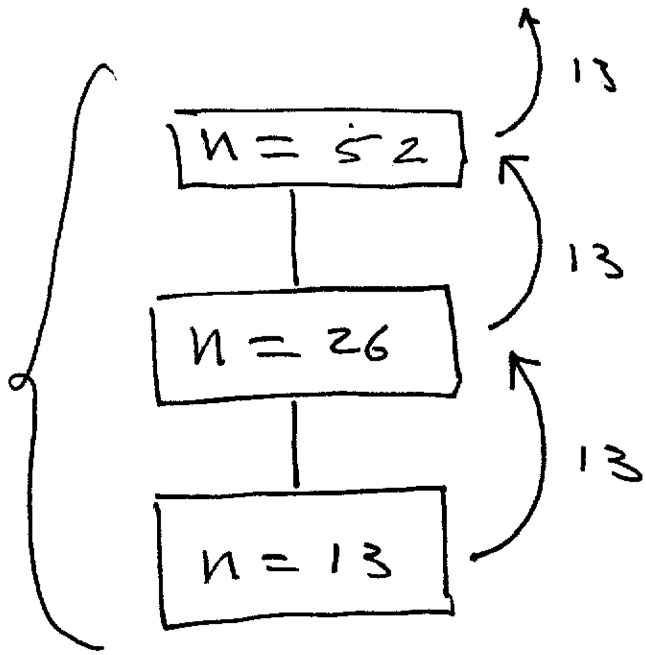
<u>Abreviated Collatz sequence:</u>

If $n = 2^k \cdot m$ where m is odd, we call m the <u>odd part</u> of n.

oddPart(n) $\longrightarrow$ returns the odd Part of n.

Ex.

box trace



$$52 = 2 \cdot 2 \cdot 13 = 2^2 \cdot 13$$

Ex. Collatz1.py

2

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Fibonacci sequence

$F_0 = 0$

$F_1 = 1$

$F_2 = 1$

$F_3 = 2$

$F_4 = 3$

$F_5 = 5$

$F_6 = 8$

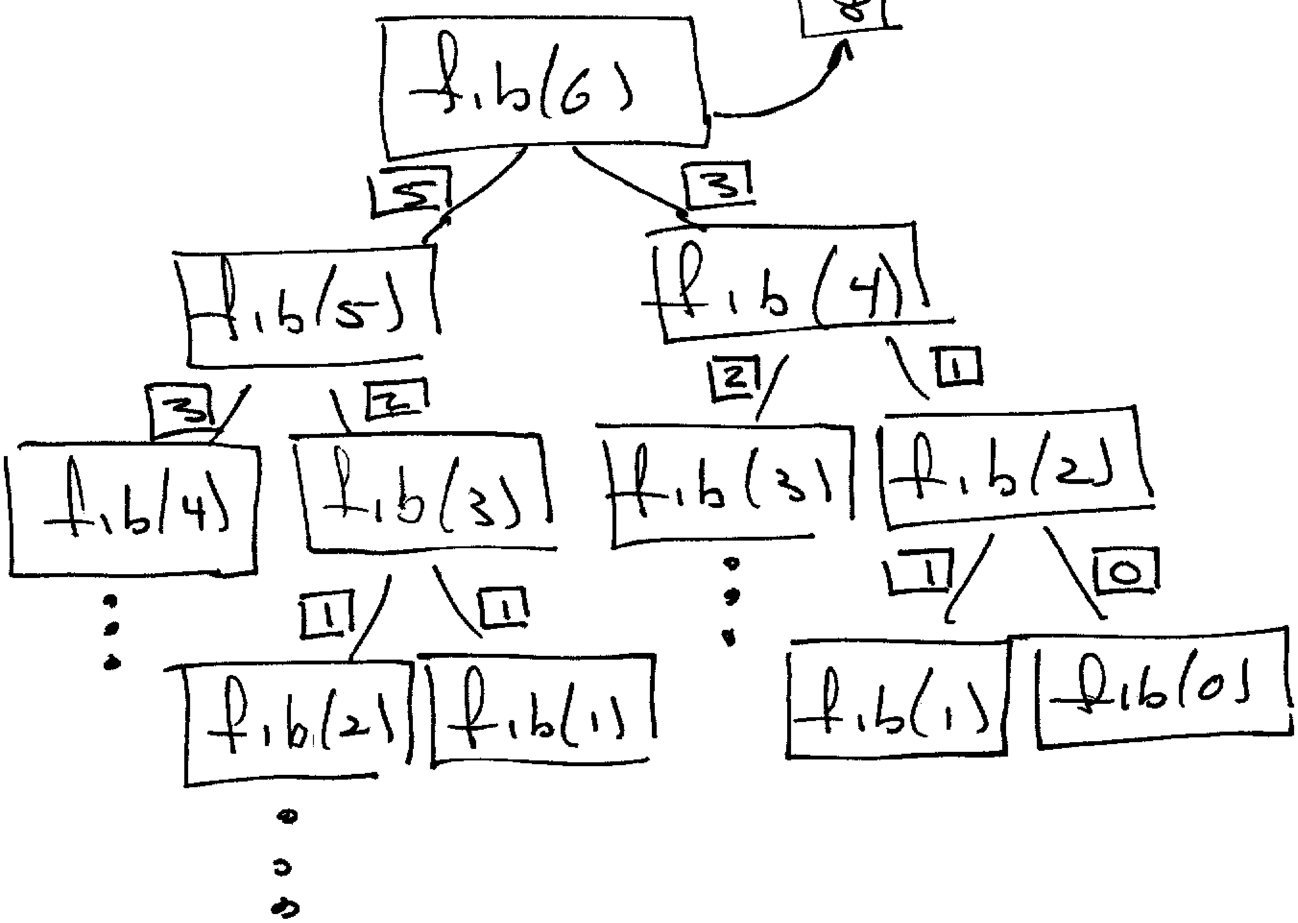
$F_7 = 13$

$\vdots$

Rule:

$$F_n = \begin{cases} 0 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ F_{n-1} + F_{n-2} & \text{if } n \geq 2 \end{cases}$$

Recursive version:



Ex. Fibonacci1.py

2

Note: if L is a list of len n.

L[-n], ---- L[-2], L[-1], L[0], L[1] ... , L[n-1]