CSE 16
Winter 2022
Quiz 2 **Solutions**

1. (15 Points) Determine whether the logical expression $(P \oplus Q) \vee (Q \Rightarrow \sim R)$ is a tautology. Either prove that it is a tautology (by any method), or show that it is not by finding an assignment of truth values to the propositional variables P, Q, R that makes the expression false. Justify your answer and the reasoning that led to it.

   Solution:
   We construct a truth table for the expression.

P	Q	R	$P \oplus Q$	$\sim R$	$Q \Rightarrow \sim R$	$(P \oplus Q) \vee (Q \Rightarrow \sim R)$
0	0	0	0	1	1	1
0	0	1	0	0	1	1
0	1	0	1	1	1	1
0	1	1	1	0	0	1
1	0	0	1	1	1	1
1	0	1	1	0	1	1
1	1	0	0	1	1	1
1	1	1	0	0	0	0

   We conclude that $(P \oplus Q) \vee (Q \Rightarrow \sim R)$ is **not a tautology**, since the assignment $\boldsymbol{P = Q = R = T}$ makes it false.

2. (20 Points) Let $K(x, y)$ be the statement "x knows y", where the universe of discourse for both x and y is the set of all people. (Assume it is possible for x to know y while y does not know x, and hence $K(x, y)$ and $K(y, x)$ are different statements.) Translate the sentence "there are two distinct persons who know each other, and no one else" into First Order Logic.

   Solution: $\exists x \, \exists y \left(x \neq y \wedge K(x,y) \wedge K(y,x) \wedge \forall z \left(\sim K(x,z) \wedge \sim K(y,z) \right) \right)$ ■

3. (15 Points) A banking system requires Personal Identification Numbers (PINs) that satisfy the following constraints.

   (1) Must consist of only decimal digits $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$.
   (2) Must have length at least 5, and at most 8.
   (3) Must not be the customer's birth date. (Here, a birth date would be encoded as a 6 digit number, so for instance May 3, 1998 would be 050398)

   How many different valid PINs are there?

   Solution:
   We partition the PINs satisfying (1) and (2) into 4 sets. By the multiplication principle we have

   length 5: # of PINs $= 10^5$
   length 6: # of PINs $= 10^6$
   length 7: # of PINs $= 10^7$
   length 8: # of PINs $= 10^8$

   There is only one PIN violating condition (3), so by the addition and subtraction principles, the number of valid PINs is: $10^5 + 10^6 + 10^7 + 10^8 - 1 = \mathbf{111,099,999}$. ■