CSE 16
Homework Assignment 6
Solutions to Selected Problems

Chapter 4 (page 126): 12
If $x \in \mathbb{R}$ and $0 < x < 4$, then $\frac{4}{x(4-x)} \geq 1$.

Proof:
Since $x < 4$ we have $4 - x > 0$. Together with $0 < x$, this gives $x(4-x) > 0$. Also

$$\begin{aligned} (x-2)^2 \geq 0 \;&\Rightarrow\; x^2 - 4x + 4 \geq 0 \\ &\Rightarrow\; 4 \geq 4x - x^2 \\ &\Rightarrow\; 4 \geq x(4-x). \end{aligned}$$

Now divide both sides by the positive number $x(4-x)$ to obtain

$$\frac{4}{x(4-x)} \geq 1.$$

As required. ■

Chapter 5 (page 136): 6
Suppose $x \in \mathbb{R}$. If $x^3 - x > 0$, then $x > -1$.

Proof:
We show that $x \leq -1 \Rightarrow x^3 - x \leq 0$, and hence $x^3 - x > 0 \Rightarrow x > -1$ by contraposition. If $x \leq -1$, it follows that $x + 1 \leq 0$. Also $x - 1 \leq -1 - 1 = -2$. Since the product of three non-positive numbers is non-positive, we have

$$\begin{aligned} &x(x-1)(x+1) \leq 0, \\ \therefore\;\; &x(x^2-1) \leq 0, \\ \therefore\;\; &x^3 - x \leq 0. \end{aligned}$$

We've shown that $x \leq -1 \Rightarrow x^3 - x \leq 0$, and therefore $x^3 - x > 0 \Rightarrow x > -1$. ■

<u>Chapter 6 (page 144): 6</u>
If $a, b \in \mathbb{Z}$, then $a^2 - 4b - 2 \neq 0$.

Proof:
Assume, to get a contradiction, that $a^2 - 4b - 2 = 0$. Then $a^2 = 4b + 2 = 2(2b + 1)$. This shows $2|a^2$, and by a result proved in class, we have $2|a$. Therefore $a = 2k$ for some $k \in \mathbb{Z}$, and

$$
\begin{aligned}
& (2k)^2 = 2(2b+1) \\
\therefore \quad & 4k^2 = 2(2b+1) \\
\therefore \quad & 2k^2 = 2b+1.
\end{aligned}
$$

But the left hand side of this equation is even, while the right hand side is odd, which is impossible. This contradiction shows our assumption was false, and therefore $a^2 - 4b - 2 \neq 0$, as claimed. ∎