CSE 16
Homework Assignment 3
Solutions to Selected Problems

<u>(2.7) 10</u>
Translate $\exists m \in \mathbb{Z}, \forall n \in \mathbb{Z}, m = n + 5$ into natural language, and determine whether it is true or false.

Solution:
Either one of the following translations work.

> **"There exists an integer $\boldsymbol{m}$ such that for all integers $\boldsymbol{n}$, we have $\boldsymbol{m = n + 5}$"**

or

> **"There is an integer that is 5 more than every integer"**

This statement is **false**. ■

<u>(3.2) 10</u>
A die is tossed four times in a row. How many different outcomes are possible.

Solution:
Each outcome is a list of length 4 from the set $A = \{1, 2, 3, 4, 5, 6\}$. Equivalently, each outcome can be identified with a string of length 4 from the alphabet A. Therefore

$$\#\text{outcomes} = |A \times A \times A \times A| = |A|^4 = 6^4 = \mathbf{1296}$$

■

(3.3) 10

Consider the lists of length six made with the symbols $\{P, R, O, F, S\}$ where repetition is allowed. How many such lists can be made if the list must end with S, and the symbol O is used more than once?

Solution:

Since position 6 is occupied by S, the remaining characters constitute a list of length 5 from the alphabet $\{P, R, O, F, S\}$, of which there are $5^5 = 3125$. Instead of counting the number of such lists that contain at least two O's, we can count the complementary set of lists, i.e those containing at most one O, and subtract that from 3125. We partition the complementary set of lists into two subsets, based on the number n of O's in the string.

(1) $n = 0$. The symbols in postions $\{1, 2, 3, 4, 5\}$ constitute a list of length 5 from the alphabet $\{P, R, F, S\}$, of which there are $4^5 = 1024$.

(2) $n = 1$. Choose which position from $\{1, 2, 3, 4, 5\}$ is to be occupied by O, then choose the remaining 4 symbols from $\{P, R, F, S\}$, with repetition allowed. By the multiplication principle, the number of ways of making these choices is $5 \cdot 4^4 = 1280$.

By the addition principle, the complementary set of lists contains $1024 + 1280 = 2304$ elements. By the subtraction principle, the number of lists with at least two O's is $3125 - 2304 = \mathbf{821}$.

Alternate Solution: (using results from section 3.5)

Our list will have 6 positions $\{1, 2, 3, 4, 5, 6\}$. Position 6 will be occupied by S, and positions $\{1, 2, 3, 4, 5\}$ will be chosen from $\{P, R, O, F, S\}$. At minimum, 2 of these must be O's. For instance:

$$\underset{1}{\underline{\text{O}}}\ \underset{2}{\underline{\text{O}}}\ \underset{3}{\underline{\ \ }}\ \underset{4}{\underline{\text{O}}}\ \underset{5}{\underline{\ \ }}\ \underset{6}{\underline{\text{S}}}\ .$$

The set of such lists can be partitioned into 4 subsets, based on the number of O's appearing in the list. This number, which we denote as n, can be 2, 3, 4 or 5.

(1) $n = 2$. Choose which 2 positions from $\{1, 2, 3, 4, 5\}$ are to be occupied by O's, then choose symbols for the remaining 3 positions from $\{P, R, F, S\}$ (where repetition is allowed). By the multiplication principle this can be done in $\binom{5}{2} \cdot 4 \cdot 4 \cdot 4 = 10 \cdot 64 = 640$ ways.

(2) $n = 3$. Choose which 3 positions from $\{1, 2, 3, 4, 5\}$ are to be occupied by O's, then choose symbols for the remaining 2 positions from $\{P, R, F, S\}$ (repetition allowed). By the multiplication principle this can be done in $\binom{5}{3} \cdot 4 \cdot 4 = 10 \cdot 16 = 160$ ways.

(3) $n = 4$. Choose which 4 positions from $\{1, 2, 3, 4, 5\}$ are to be occupied by O's, then choose a symbol for the remaining 1 position from $\{P, R, F, S\}$ (repetition allowed). By the multiplication principle this can be done in $\binom{5}{4} \cdot 4 = 5 \cdot 4 = 20$ ways.

(4) $n = 5$. Let all 5 positions from $\{1, 2, 3, 4, 5\}$ be occupied by O's. This can be done in just 1 way.

By the addition principle, there are $640 + 160 + 20 + 1 = \mathbf{821}$ such lists. ∎