CSE 16
Homework Assignment 2
Solutions to Selected Problems

(1.8) 14

For each $\alpha \in I$, let A_α be a set. Suppose that $\emptyset \neq J \subseteq I$. It follows that

$$\bigcap_{\alpha \in I} A_\alpha \subseteq \bigcap_{\alpha \in J} A_\alpha$$

Solution:

Let x be an element of the left hand side (LHS). Then $x \in A_\alpha$ for all $\alpha \in I$. Since $J \subseteq I$, this implies $x \in A_\alpha$ for all $\alpha \in J$. Therefore x is also a member of the right hand side (RHS), and hence LHS ⊆ RHS. ■

(2.5) 10

Suppose the statement $\big((P \wedge Q) \vee R\big) \Rightarrow (R \vee S)$ is false. Find the truth values of P, Q, R and S. (Note this can be done without constructing a truth table.)

Solution:

The only way for an implication to be false is for the hypothesis to be true while the conclusion is false. Thus $(P \wedge Q) \vee R$ is true, and $R \vee S$ is false. Since $R \vee S$ is false, both operands must be false, i.e.

$$R = S = F.$$

Since R is false and $(P \wedge Q) \vee R$ is true, we must have $P \wedge Q$ true, hence both operands are true, i.e.

$$P = Q = T.$$

Thus if $\big((P \wedge Q) \vee R\big) \Rightarrow (R \vee S)$ is false, then $\boldsymbol{P = Q = T}$**, and** $\boldsymbol{R = S = F}$. ■

<u>(2.6) 10</u>

Decide whether $(P \Rightarrow Q) \vee R$ and $\sim\big((P \wedge \sim Q) \wedge \sim R\big)$ are logically equivalent.

Solution:

P	Q	R	$P \Rightarrow Q$	$(P \Rightarrow Q) \vee R$	$\sim Q$	$P \wedge \sim Q$	$\sim R$	$(P \wedge \sim Q) \wedge \sim R$	$\sim\big((P \wedge \sim Q) \wedge \sim R\big)$
0	0	0	1	1	1	0	1	0	1
0	0	1	1	1	1	0	0	0	1
0	1	0	1	1	0	0	1	0	1
0	1	1	1	1	0	0	0	0	1
1	0	0	0	0	1	1	1	1	0
1	0	1	0	1	1	1	0	0	1
1	1	0	1	1	0	0	1	0	1
1	1	1	1	1	0	0	0	0	1

Since the two expressions have the same truth table, they are **logically equivalent**. ■

Alternate Solution:

Using logical identities, we have

$$\begin{aligned} \sim\big((P \wedge \sim Q) \wedge \sim R\big) &= \sim(P \wedge \sim Q) \vee \sim(\sim R) && \text{DeMorgan} \\ &= \big(\sim P \vee \sim(\sim Q)\big) \vee R && \text{DeMorgan \& Double Negation} \\ &= (\sim P \vee Q) \vee R && \text{Double Negation} \\ &= (P \Rightarrow Q) \vee R && \text{Implication} \end{aligned}$$

It follows that $(P \Rightarrow Q) \vee R = \sim\big((P \wedge \sim Q) \wedge \sim R\big)$, so again the expressions are **logically equivalent**. ■