

CSE 16

Homework Assignment 1

Solutions to Selected Problems

(1.2) 2g

Let $A = \{\pi, e, 0\}$ and $B = \{0, 1\}$. Then

$$B \times B = \{ (0,0), (0,1), (1,0), (1,1) \}$$

and therefore

$$A \times (B \times B) = \{ (\pi, (0,0)), (\pi, (0,1)), (\pi, (1,0)), (\pi, (1,1)), \\ (e, (0,0)), (e, (0,1)), (e, (1,0)), (e, (1,1)), \\ (0, (0,0)), (0, (0,1)), (0, (1,0)), (0, (1,1)) \}$$

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Let $S = \{ \{0, 1\}, \{0, 1, \{2\}\}, \{0\} \}$. Then

$$\mathbb{P}(S) =$$

$$\{ \emptyset, \{\{0, 1\}\}, \{\{0, 1, \{2\}\}\}, \{\{0\}\}, \{\{0, 1\}, \{0, 1, \{2\}\}\}, \{\{0, 1\}, \{0\}\}, \{\{0, 1, \{2\}\}, \{0\}\}, \{\{0, 1\}, \{0, 1, \{2\}\}, \{0\}\} \}$$

Note to graders: the outer braces are not necessary for full credit, since the problem said "list all subsets", not "determine the power set".

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Let $|A| = m$. Then $|\mathbb{P}(A)| = 2^m$, and number of 1-element subsets of $\mathbb{P}(A)$ is therefore also 2^m . There is only one 0-element subset of $\mathbb{P}(A)$, namely $\emptyset$. Thus

$$\{ X \subseteq \mathbb{P}(A) : |X| \leq 1 \} = \{ X \subseteq \mathbb{P}(A) : |X| = 1 \} \cup \{ X \subseteq \mathbb{P}(A) : |X| = 0 \}$$

and hence

$$|\{ X \subseteq \mathbb{P}(A) : |X| \leq 1 \}| = |\{ X \subseteq \mathbb{P}(A) : |X| = 1 \}| + |\{ X \subseteq \mathbb{P}(A) : |X| = 0 \}| \\ = 2^m + 1$$