cse 16 1-4-22

- Important note: Going forward we will meet $\boxed{\text{11:30 am to 1:05 pm}}$

Why do we prove things in Engineering?

- Ultimate explanation.
- Proof in a formal essay.
- clear thinking.

chap 1. sets

(1.1)

Naive definition:

A set is an (unordered) collection of any objects whatever.

The objects in a set are called **members**, or **elements**.

notation: $x \in S$

Ex. list members in $\{\ldots\}$

- $\{1, 2, 3\}$
- $\{1, 2, 3, \ldots, 99, 100\}$
- $\{1, 2, 3, \ldots\ldots\}$

Important sets

- natural numbers:

$$\mathbb{N} = \{1, 2, 3, 4, \ldots\ldots\} \ (= \mathbb{Z}^+)$$

- integers

$$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$$

- Rational numbers : $\mathbb{Q}$
- Real numbers : $\mathbb{R}$
- Complex numbers : $\mathbb{C}$
- empty set:

$$\emptyset = \{\ \} \quad \text{no elements.}$$

Two sets are equal iff they have same members.

$\{1, 2, 3\} = \{2, 3, 1\}$
$= \{1, 2, 1, 1, 2, 3, 2, 3, 3, 2, 1\}$

<u>Set Builder Notation</u>

"such that"

$A = \{\text{expression} \mid \text{Property}\}$

or $A = \{\text{exp} \in \text{set} \mid \text{Property}\}$

existing set

<u>Ex.</u> $\{x \mid x \text{ is a Prime number}\}$

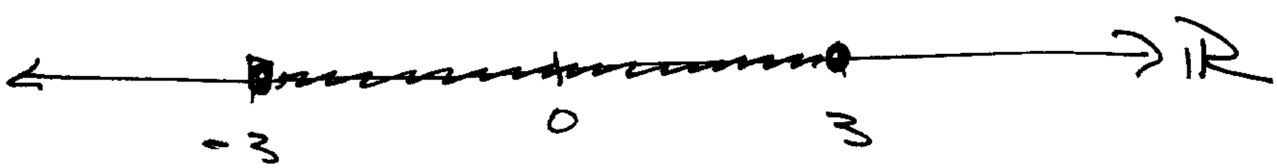
$= \{2, 3, 5, 7, 11, 13, 17, 19, \ldots\}$

Ex. same set

$$\{n \in \mathbb{N} \mid n \text{ is prime}\}$$

Ex. $\{n \in \mathbb{Z} \mid |n| \le 3\} = \{-3, -2, -1, 0, 1, 2, 3\}$

Ex. $\{x \in \mathbb{R} \mid |x| \le 3\} = [-3, 3]$



Ex. $\mathbb{Q} = \left\{ \frac{p}{q} \mid p, q \in \mathbb{Z} \text{ and } q \neq 0 \right\}$

Defn

A set S is called finite if it contains exactly n elements for $n \in \underbrace{\{0, 1, 2, 3, \dots\}}_{\text{non-negative integers.}}$

notation: $\underbrace{|S|}_{\text{cardinality of } S} = n$

If S is not finite, its called <u>infinite</u>. write: $|S| = \infty$

(1.2) Cartesian Products

An <u>ordered n-tuple</u> is a collection of n objects that is ordered.

<u>Notation</u>: $(x_1, x_2, x_3, \ldots, x_n)$

If $n=2$: ordered Pair (x_1, x_2)

$n=3$: ordered triple (x_1, x_2, x_3)

<u>note</u>: $(x_1, x_2) = (y_1, y_2)$ iff both $x_1 = y_1$ and $x_2 = y_2$.

"if and only if"

<u>Defn</u>

The <u>Cartesian Product</u> of two sets A, B is the set of all ordered pairs (x, y) where $x \in A$, $y \in B$.

<u>notation</u>: $A \times B$

Thus: $A \times B = \{(x,y) \mid x \in A \text{ and } y \in B\}$

<u>Ex.</u> $A = \{1, 2\}$, $B = \{3, 4, 5\}$

$$A \times B = \{(1,3), (1,4), (1,5), (2,3), (2,4), (2,5)\}$$

<u>Theorem</u>
If A, B are finite sets, then $A \times B$ is finite and

$$|A \times B| = |A| \cdot |B|.$$

more generally, the <u>Cartesian Product</u> of $A_1, A_2, \ldots, A_n$

$$A_1 \times A_2 \times \cdots \times A_n$$

$$= \{(x_1, x_2, \ldots, x_n) \mid x_i \in A_i \text{ for } 1 \le i \le n\}$$

<u>Theorem</u>
If all A_i are finite, so is $A_1 \times A_2 \times \cdots \times A_n$ and

$$|A_1 \times A_2 \times \cdots \times A_n| = |A_1| \cdot |A_2| \cdots |A_n| .$$

(1.3) Subsets

Defn

We say a set A is a subset of a set B iff every member of A is also a member of B.

notation: $A \subseteq B$

We say A is a proper subset of B iff $A \subseteq B$ and $A \neq B$.

notation: $A \subsetneq B$

other notation: $A \subset B$
Don't use this since its ambiguous

<u>note</u>: for any set S:

- $S \subseteq S$
- $\phi \subseteq S$

<u>note</u>: 'contains', 'in'

- $x \in S$: x is in S, S contains x.
- $A \subseteq S$: A is in S, S contains A.

<u>Ex.</u> $\{\{1,2\},3\} \neq \{1,2,3\}$