cse 16 1-20-22

(2.8) Quantifiers & conditionals

Ex. (algebra class)

$$x^2 - y^2 = (x-y)(x+y)$$

what the author means is

$$\underbrace{\forall x \in \mathbb{R}, \forall y \in \mathbb{R}}_{}: x^2 - y^2 = (x-y)(x+y)$$

often given implicitly!

Ex. (book)

$$\underbrace{\ldots\ldots}_{\forall x \in \mathbb{Z}} (x \text{ a mult. of } 6) \Rightarrow (x \text{ is even})$$

Advice: always make quantifiers explicit.

(2.9) Translation

Consider the stmt.

'all horses in CA are blue'

Ex. $U = \{\text{horses in CA}\}$

$B(x) =$ 'x is blue'

Translation : $\forall x\, B(x)$

Ex. $U = \{\text{horses}\}$

$C(x) =$ 'x lives in CA'

Translation : $\forall x\, (C(x) \Rightarrow B(x))$

Ex $U = \{\text{living things}\}$

$H(x) =$ ' x is a horse '

Translation: $\forall x \left(H(x) \wedge C(x) \Longrightarrow B(x) \right)$

Note: $T \Rightarrow P = P$ is a logical equiv.

P	$T \rightarrow P$
0	0
1	1

$P = (T \rightarrow P)$

$\therefore$ first example

$$\forall x \left(T \Rightarrow B(x) \right)$$

Thus if $S \subseteq U$, then

$$\forall x \in U \; (x \in S \rightarrow P(x))$$

is equiv. to

$$\forall x \in S : P(x)$$

Ex. $L(x,y) =$ 'x loves y'

$U = \{\text{people}\}$

a) 'everybody loves somebody'

$$\forall x \; \exists y : L(x,y)$$

b) 'somebody loves everybody'

$$\exists x \, \forall y \; L(x, y)$$

c) $\exists y \, \forall x : L(x, y)$ says

'there is a person who is loved by all'

d) $\forall y \, \exists x : L(x, y)$ says

'everyone is loved by someone'

e) 'there is exactly one person who everyone loves'

$$\exists y \left(\forall x \; L(x,y) \wedge \forall z \left((z \neq y) \rightarrow \exists w \sim L(w,z) \right) \right)$$

f.) 'there is a person who loves everyone but themself'

$$\exists x \left(\sim L(x,x) \wedge \forall y \, ((y \neq x) \rightarrow L(x,y)) \right)$$

(2.10) Negation

To better understand $\forall$, $\exists$, consider the finite universe

$$\mathcal{U} = \{1, 2\}$$

let $P(x)$ be a prop. fcn. on $\mathcal{U}$.

- $\forall x \, P(x) = P(1) \wedge P(2)$
- $\exists x \, P(x) = P(1) \vee P(2)$

Therefore

- $\sim \forall x P(x) = \sim (P(1) \wedge P(2))$

$$= \sim P(1) \vee \sim P(2)$$

$$= \exists x \sim P(x)$$

- $\sim \exists x P(x) = \sim (P(1) \vee P(2))$

$$= \sim P(1) \wedge \sim P(2)$$

$$= \forall x \sim P(x)$$

In fact these identities hold over any universe

$$\boxed{\begin{array}{c} \sim \forall x P(x) = \exists x \sim P(x) \\ \hline \sim \exists x P(x) = \forall x \sim P(x) \end{array}}$$

stmt	true when	false when
$\forall x\, P(x)$	$P(x)$ true for all $x \in U$	$P(x)$ is false for at least one $x \in U$.
$\exists x\, P(x)$	$P(x)$ is true for at least one $x \in U$	$P(x)$ is false for all $x \in U$.

(2.11) Inference Rules.

Read this section.

Chap 3 : Combinatorics

(3.1) lists

Defn

A list (or sequence) is an ordered collection of objects (possibly with repetition.

The objects in a list are called entries, element, terms or components.

The number of elements is its the length of the list, which may be ∞.

notation: $(1, 3, 2, 2, 1)$

empty list: $(\ )$

<u>Defn</u>

A <u>string</u> is a sequence of symbols from some alphabet, written without parens or commas

<u>Ex.</u> alpha $\{a, b, c\}$

string: accabbaacbaca

(3.2) Multiplication Principle

Ex.

find # of lists (x, y, z) where
$x \in \{1,2,3\}$, $y \in \{3,4\}$, $z \in \{4,5\}$

$(1, 3, 4)$

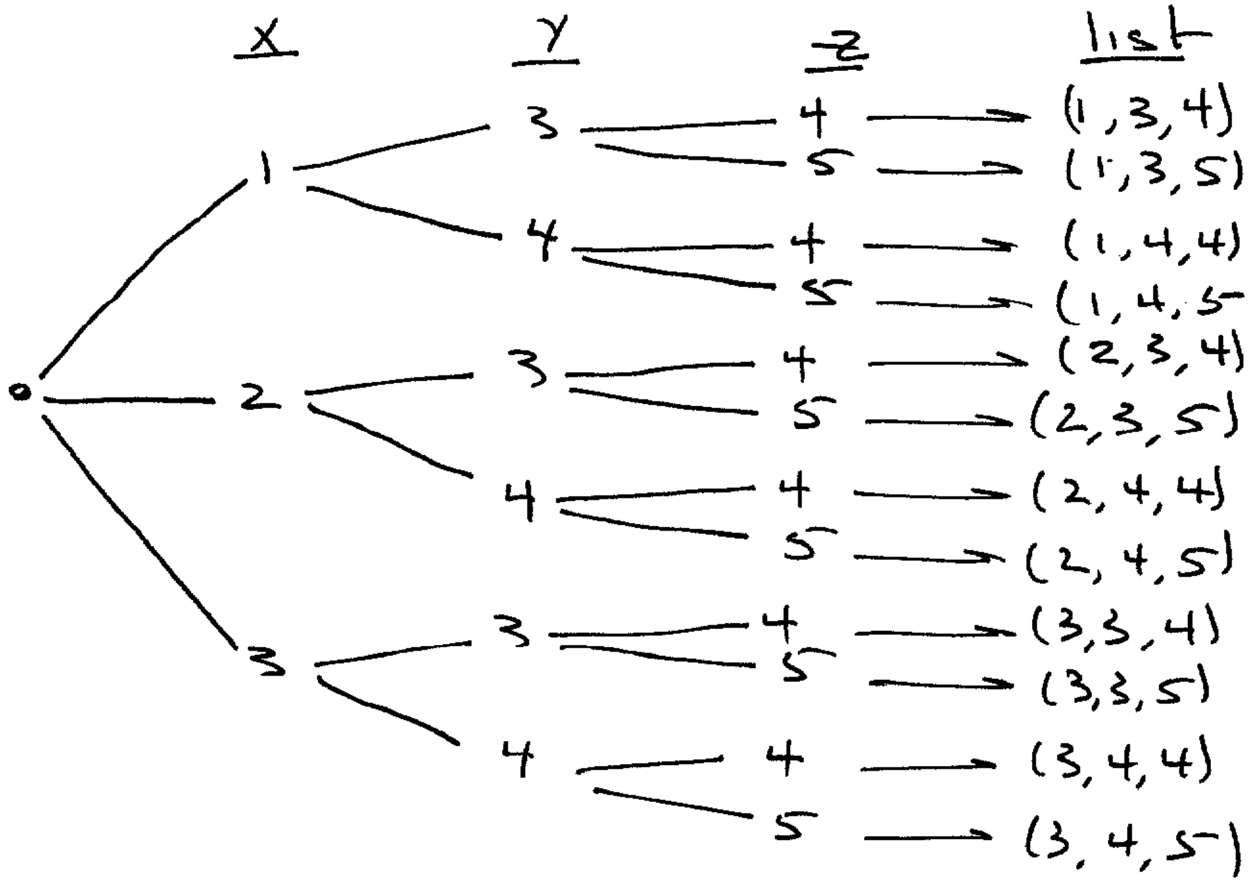
$(3, 3, 5)$

$(2, 4, 4)$

$\vdots$

more.

To be systematic, use a tree diagram.



$$\#\text{lists} = \boxed{12} = 3\cdot 2\cdot 2$$

In general:

$$\Big|\underbrace{\{(x,y,z) \mid x\in A \wedge y\in B \wedge z\in C\}}_{\text{this set is } A\times B\times C}\Big| = |A|\cdot|B|\cdot|C|$$

i.e. $|A\times B\times C| = |A|\cdot|B|\cdot|C|$

Ex. find # of strings of length 3 from $\{a, b, c\}$.

①	②	③
b	c	c

$$\# \text{ choices} = 3 \cdot 3 \cdot 3 = \boxed{27}$$

Ex.
find the # of such strings in which no letter is repeated

1	2	3	string
a	b	c	abc
	c	b	acb
b	a	c	bac
	c	a	bca
c	a	b	cab
	b	a	cba

$$\# \text{ choices} = 3 \cdot 2 \cdot 1 = \boxed{6}$$

Multiplication Principle

suppose in forming a list $(x_1, x_2, \ldots, x_n)$ that there are

a_1 choices for x_1
a_2 " " x_2
a_3 " " x_3
$\vdots$
a_n " " x_n

Then the number of such lists is $\boxed{a_1 \cdot a_2 \cdot a_3 \cdots a_n}$

note: in general the # of choices a_i for x_i may depend on preceding choices

$$(x_1, x_2, x_3, \ldots, x_{i-1}, x_i, \ldots\ldots)$$

where $2 \leq i \leq n$

Ex. Bit strings

a bit string is a string over alphabet $\{0, 1\}$.

$$\boxed{(\text{\# of bit strings of len. } n) = 2^n}$$

① ② ③ ... ⓝ

1 1 0 1

$$\#\text{choices} = 2 \cdot 2 \cdot 2 \cdots\cdots 2 = 2^n$$

Ex. The # of strings of length n from an alphabet of size a is $\boxed{a^n}$

① ② ... ⓝ

$$\#\text{choices}: \quad a \quad a \quad \cdots \quad a = \boxed{a^n}$$

Ex. The number of such strings with no repetition is

$$\begin{array}{ccccc} ① & ② & ③ & & ⓝ \\ \underline{\cdot} & \underline{\cdot} & \underline{\cdot} & & \underline{\cdot} \end{array}$$

$$\#\text{choices} = a \ (a-1) \ (a-2) \cdots\cdots (a-n+1)$$

$$= a\cdot(a-1)(a-2)\cdots(a-n+1)$$

$$= \boxed{\frac{a!}{(a-n)!}}$$

(2.3) Addition & Subtraction Principles

Ex find # of strings of len. 4 from $\{a, b, c, d, e, f\}$ satisfy.

- no repeated letters
- must contain a

There are 4 types of such strings

I. $\underline{a}\ \underset{5}{\underline{\quad}}\ \underset{4}{\underline{\quad}}\ \underset{3}{\underline{\quad}}$ $\qquad 5 \cdot 4 \cdot 3 = 60 \ +$

II. $\underset{5}{\underline{\quad}}\ \underline{a}\ \underset{4}{\underline{\quad}}\ \underset{3}{\underline{\quad}}$ $\qquad \ldots = 60 \ +$

III. $\underset{5}{\underline{\quad}}\ \underset{4}{\underline{\quad}}\ \underline{a}\ \underset{3}{\underline{\quad}}$ $\qquad \ldots = 60 \ +$

IV. $\underset{5}{\underline{\quad}}\ \underset{4}{\underline{\quad}}\ \underset{3}{\underline{\quad}}\ \underline{a}$ $\qquad \ldots = 60 \ +$

$\boxed{240}$