cse 16 1-13-22

- quiz 1 : today
- hw1 : reviews underway
- hw2 : posted, opens Sunday

(2.2) logical operations: and, or, not

and (conjunction) : $p \wedge q$

p	q	$p \wedge q$
F	F	F
F	T	F
T	F	F
T	T	T

True only in case both operands true, false otherwise.

Ex. p = 'it is raining today'
q = 'today is Wednesday'

or (disjunction, inclusive or) : $p \vee q$

p	q	$p \vee q$
F	F	F
F	T	T
T	F	T
T	T	T

False only in case both operands are false, otherwise true

Ex. p = ... same ..

q = .. " ..

exor (exclusive or) : $p \oplus q$

p	q	$p \oplus q$
F	F	F
F	T	T
T	F	T
T	T	F

True when operands have opposite truth values, otherwise false.

Ex. 'your money or your life'

not (negation): $\sim p$ $(\neg p)$

p	$\sim p$
F	T
T	F

reverse truth value of operand

(2.3) operation: implication, conditional

conditional (implication):

notation $p \Rightarrow q$ or $p \longrightarrow q$

hypothesis conclusion

p	q	$p \Rightarrow q$
F	F	T
F	T	T
T	F	F
T	T	T

False only in case hypothesis true and conclusion false, true otherwise.

Read this : $p \to q$ as

- 'if p, then q'
- 'p implies q'
- 'p only if q'
- 'p is sufficient for q'
- 'q, if p'
- 'q whenever p'
- 'q is necessary for p'

<u>Ex.</u>

p = 'it is sunny today'

q = 'I will take you to the beach'

Related propositions to $p \to q$:

- converse of $p \to q$ is $q \to p$
- contrapositive of $p \to q$ is $\sim q \to \sim p$
- reverse of $p \to q$ is $\sim p \to \sim q$

note: $\sim$ has highest precedence, so

$$\sim p \rightarrow q \quad \text{means} \quad (\sim p) \rightarrow q$$

$$\sim p \vee q \quad \text{"} \quad (\sim p) \vee q$$

(2.4) OP : biconditional

biconditional : $p \Leftrightarrow q$ (or $p \leftrightarrow q$)

p	q	$p \Leftrightarrow q$
F	F	T
F	T	F
T	F	F
T	T	T

True when operands have same truth value, false otherwise

Read as: 'p if and only if q'
'p is necessary and sufficient for q'
'p is equivalent to q'

(2.5) Truth tables

- Propositional variables: $\underbrace{p, q, r, s \ldots}_{\text{atomic statement}}$

- Compound statements are atomic statements, and all statements built from these using logical operators.

Ex. $(p \leftrightarrow q) \vee (\sim q \rightarrow r)$

p	q	r	$p \leftrightarrow q$	$\sim q$	$\sim q \rightarrow r$	$(p \leftrightarrow q) \vee (\sim q \rightarrow r)$
0	0	0	1	1	0	1
0	0	1	1	1	1	1
0	1	0	0	0	1	1
0	1	1	0	0	1	1
1	0	0	0	1	0	0
1	0	1	0	1	1	1
1	1	0	1	0	1	1
1	1	1	1	0	1	1

Ex. $\sim(p \vee q)$

p	q	$p \vee q$	$\sim(p \vee q)$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

Ex. $p \to q$ not same as $q \to p$

p	q	$p \to q$	$q \to p$
0	0	1	1
0	1	1	0
1	0	0	1
1	1	1	1

Ex. $p \to (q \to r)$ vs. $(p \to q) \to r$

p	q	r	$q \to r$	$p \to (q \to r)$	$p \to q$	$(p \to q) \to r$
0	0	0	1	1	1	0
0	0	1	1	1	1	1
0	1	0	0	1	1	0
0	1	1	1	1	1	1
1	0	0	1	1	0	1
1	0	1	1	1	0	1
1	1	0	0	0	1	0
1	1	1	1	1	1	1

different

Exercise

check $(p \vee q) \wedge r$ is different from $p \vee (q \wedge r)$. Therefore $p \vee q \wedge r$ makes no sense!

(2.6) logical equivalence

Defn

A <u>tautology</u> is a compound statement that is true for <u>any</u> assignment of truth values to its prop. variables.

Ex. $p \vee \sim p$ is a tautology

p	$\sim p$	$p \vee \sim p$
0	1	1
1	0	1

Ex. $p \rightarrow p$ is a tautology

p	$p \rightarrow p$
0	1
1	1

Ex. $p \Rightarrow (p \vee q)$ is a tautology.

p	q	$p \vee q$	$p \Rightarrow (p \vee q)$
0	0	0	1
0	1	1	1
1	0	1	1
1	1	1	1

Defn

a contradiction is a compound stmt, that is false for any assignment of truth values to its prop. variables. (also called: unsatisfiable)

Ex. $p \wedge \sim p$

p	$\sim p$	$p \wedge \sim p$
0	1	0
1	0	0

<u>Defn</u>

a stmt. is called <u>satisfiable</u> iff it is not a contradiction. (i.e. its truth table has at least one T in it.)

<u>Defn</u>

a statement is called a <u>contingency</u> iff it is neither a tautology nor a contradiction.

<u>Defn</u>

Two compound stmts X, Y are said to be logically equivalent iff the stmt. $X \leftrightarrow Y$ is a tautology.

<u>notation</u>: $X \equiv Y$

equiv. defn

X, Y are logically equivalent iff their truth tables are identical.

Rmk This is what is meant by = in algebra: $a(b+c) = ab + ac$

Ex. $P \Rightarrow q = \sim P \vee q$ ✓

show $(P \Rightarrow q) \Leftrightarrow (\sim P \vee q)$ is a tautology.

P	q	$P \Rightarrow q$	$\sim P$	$\sim P \vee q$	$(P \Rightarrow q) \Leftrightarrow (\sim P \vee q)$
0	0	1	1	1	1
0	1	1	1	1	1
1	0	0	0	0	1
1	1	1	0	1	1

same

Exercise :

- $p \Rightarrow q \;=\; \sim q \Rightarrow \sim p$
- $p \Rightarrow q \;\neq\; q \Rightarrow p$
- all identities on p. 52 of B.O.P.