CSE 16
Lab Assignment 4

The *Hereditarily Finite Sets* are the finite sets that can be constructed from "nothing". Starting with the empty set (denoted here by `0`), we form the following sequence of sets.

n (decimal)	n (binary)	$H(n)$
0	0	`0`
1	1	`{0}`
2	10	`{{0}}`
3	11	`{0, {0}}`
4	100	`{{{0}}}`
5	101	`{0, {{0}}}`
6	110	`{{0}, {{0}}}`
7	111	`{0, {0}, {{0}}}`
8	1000	`{{0, {0}}}`
9	1001	`{0, {0, {0}}}`
10	1010	`{{0}, {0, {0}}}`
11	1011	`{0, {0}, {0, {0}}}`
12	1100	`{{{0}}, {0, {0}}}`
13	1101	`{0, {{0}}, {0, {0}}}`
14	1110	`{{0}, {{0}}, {0, {0}}}`
15	1111	`{0, {0}, {{0}}, {0, {0}}}`
16	10000	`{{{{0}}}}`

The initial term is $H(0) =$ `0`, the empty set. Observe that each subsequent term in the sequence contains, as its members, certain earlier terms in the sequence. The n^{th} term $H(n)$ can be obtained recursively by expressing n in binary, then finding those positions i in the corresponding binary numeral that are occupied by 1's. The elements of $H(n)$ are those sets $H(i)$.

For instance, we obtain $H(5)$ by reading the bit string 101 from right to left (least to most significant). The 1's are in positions 0 and 2, so we include term 0, exclude term 1, and include term 2. Thus

$$H(5) = \{H(0), H(2)\} = \texttt{\{0, \{\{0\}\}\}}$$

We compute $H(13)$ in slightly more detail. The binary numeral for 13 is $(1101)_2$. Labeling the positions in this string from right to left, we have

bit:	1	1	0	1
position:	3	2	1	0

Since the 1's are in positions 0, 2, and 3 we have

$$H(13) = \{H(0), H(2), H(3)\} = \texttt{\{0, \{\{0\}\}, \{0, \{0\}\}\}}$$

In general, if $x = (x_k, x_{k-1}, \ldots, x_1, x_0)$ is the bit string representing n in base-2, then

$$H(n) = \{\, H(i) \mid 0 \leq i \leq k \text{ and } x_i = 1 \,\}$$

Your goal in this project will be to print out $H(n)$ for all n in the range $0 \leq n \leq 500$. There are basically two methods for accomplishing this. The top-down method is to compute $H(n)$ recursively as described above. First express n as a bit string, then recursively compute $H(i)$ for each position i in the bit string for n which contains 1. The base of the recursion is the empty set $H(0) = \mathtt{0}$.

The bottom-up method is to produce binary numerals for the numbers 0 through n, initialize $H(0) = \mathtt{0}$, then proceed to compute $H(i)$ for $i = 1$ to n by examining the binary numeral for i, and previously computed terms.

There may of course be other methods. You will turn in a file called `lab4.txt` containing a brief paragraph describing your method, then the sets $H(n)$ for $0 \leq n \leq 500$. We list the sets $H(0)$ through $H(20)$ below with the required formatting.

```
0
{0}
{{0}}
{0, {0}}
{{{0}}}
{0, {{0}}}
{{0}, {{0}}}
{0, {0}, {{0}}}
{{0, {0}}}
{0, {0, {0}}}
{{0}, {0, {0}}}
{0, {0}, {0, {0}}}
{{{0}}, {0, {0}}}
{0, {{0}}, {0, {0}}}
{{0}, {{0}}, {0, {0}}}
{0, {0}, {{0}}, {0, {0}}}
{{{{0}}}}
{0, {{{0}}}}
{{0}, {{{0}}}}
{0, {0}, {{{0}}}}
{{{0}}, {{{0}}}}
```

Observe that each element, other than $H(0)$, appears as a comma separated list of prior terms, enclosed in curly braces, with a space after each comma. You will represent the empty set as the number 0. We have presented it in this document as 0 in the `consolas` font: `0`. **You are not responsible for the font** since you are turning in a plain text file which contains no such meta data. Thus your listing will contain the characters: `0`, `{`, `}`, `comma`, `space`, `newline`, and nothing else. In particular, you will not label each line with a base-10 numeral.

Clearly this last lab assignment for CSE 16 is the most challenging of the quarter. Submit the file `lab4.txt` in the usual manner on Gradescope by the due date. As usual, start early and ask plenty of questions.