

## CSE 16

### Lab Assignment 3

In this assignment, you will print all permutations of the 6-element set  $\{1, 2, 3, 4, 5, 6\}$ . Furthermore, you will print them in their natural order. Illustrating on a smaller example, the  $3! = 6$  permutations of  $\{1, 2, 3\}$  are

123  
132  
213  
231  
312  
321

Notice that if we consider each permutation as a 3-digit number, then the above list is sorted in increasing order. This is what is meant by the natural order, also called *lexicographic order*, or *alphabetical order*. Doing the same thing with the set  $\{1, 2, 3, 4\}$ , we get the  $4! = 24$  permutations

1234  
1243  
1324  
1342  
1423  
1432  
2134  
2143  
2314  
2341  
2413  
2431  
3124  
3142  
3214  
3241  
3412  
3421  
4123  
4132  
4213  
4231  
4312  
4321

Your task is to print out a similarly sorted list of permutations for the 6-element set  $\{1, 2, 3, 4, 5, 6\}$ . There are many ways to systematically produce all permutations of a set in the lexicographic order, and you are free to use any method you like for this project. One such method is presented here.

The following algorithm takes as input a permutation  $A = (A_1, A_2, A_3, \dots, A_n)$  of the set  $\{1, 2, 3, \dots, n\}$ , then alters the sequence to be the next permutation in lexicographic order. If the input is the last

permutation  $(n, n - 1, n - 2, \dots, 1)$ , the algorithm goes back to the beginning of the sequence, to the first permutation  $(1, 2, 3, \dots, n)$ .

### nextPermutation( $A$ )

```
scan the permutation  $A$  from right-to-left starting at  $A_{n-1}$ 
    if the current element is less than its right-hand neighbor
        call the current element the pivot
        stop scanning
if the left end was reached without finding a pivot
    reverse  $A$  (permutation was lexicographically last, so start over)
    return
scan the permutation  $A$  from right-to-left again, this time starting at  $A_n$ 
    if the current element is larger than the pivot
        call the current element the successor
        stop scanning
swap the pivot and the successor elements
reverse the portion of  $A$  to the right of the pivot's original location
return
```

Let us illustrate on the permutation  $(8, 5, 4, 3, 9, 7, 6, 2, 1)$  of the 9-element set  $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ . We first find the pivot and successor

$(8, 5, 4, 3, 9, 7, 6, 2, 1)$   
          ↑      ↑  
     Pivot  Successor

then swap pivot and successor

$(8, 5, 4, 6, 9, 7, 3, 2, 1)$   
          ↑      ↑  
     Successor  Pivot

and then reverse everything to the right of where the pivot was originally located (i.e. the successor's current position).

$(8, 5, 4, 6, 1, 2, 3, 7, 9)$

Thus the lexicographically next permutation after  $(8, 5, 4, 3, 9, 7, 6, 2, 1)$  is  $(8, 5, 4, 6, 1, 2, 3, 7, 9)$ . Using this algorithm in a loop, one can produce, in order, all permutations of a set.

As mentioned, there are other methods for producing the required permutations. You will turn in a file called `lab3.txt` containing a short description of your method, followed by a blank line, then followed by all permutations of  $\{1, 2, 3, 4, 5, 6\}$  in lexicographic order, each on a single line with no commas or spaces. The first permutation will be printed as 123456, and the last appearing as 654321.

Submit `lab3.txt` in the usual manner on Gradescope by the due date. As usual, start early and ask plenty of questions.