CSE 16 8-8-24

- Quiz 5 Today: 6.1 - 6.4
- SETs opens today (time?)

  If response rate $\geq 85\%$, I will add 0.5% to each student's overall score.

<u>Recall</u>

$$\underbrace{\binom{n}{n_1, n_2, \ldots, n_k}}_{\text{multinomial coefficient}} = \frac{n!}{n_1!\, n_2! \cdots n_k!}$$

observe

$$\binom{n}{k,\ n-k} = \frac{n!}{k!(n-k)!} = \binom{n}{k}$$

also

$$\binom{n}{a,\ b,\ n-a-b} = \binom{n}{a} \cdot \binom{n-a}{b}$$

check:

$$\binom{n}{a,\ b,\ n-a-b} = \frac{n!}{a!\, b!\, (n-a-b)!}$$

$$= \frac{n!}{a!(n-a)!} \cdot \frac{(n-a)!}{b!(n-a-b)!}$$

$$= \binom{n}{a} \cdot \binom{n-a}{b}$$

Multinomial Theorem

Let $x_1, x_2, \ldots, x_k \in \mathbb{R}$. Then

$$(x_1+x_2+\cdots+x_k)^n = \sum_{n_1+\cdots+n_k=n} \binom{n}{n_1,\ldots n_k} x_1^{n_1}\cdot x_2^{n_2}\cdots x_k^{n_k}$$

where the sum is over all k-tuples $(n_1,\ldots,n_k)$ satisfying $n_1+\cdots+n_k=n$, and $n_i \in \mathbb{N}$ $(1\le i\le k)$.

Exercise

Prove this by imitating the combinatorial proof of the Binomial thm.

Exercise

How many terms are there in the RHS of last thm.?

i.e. how many solutions to

$$n_1 + n_2 + \cdots + n_k = n$$

are there?

Exercise

Prove that

$$\sum_{n_1+\cdots+n_k=n} \binom{n}{n_1, \ldots, n_k} = k^n$$

7.1 Intro to Discrete Probability

Definitions

- an *experiment* is a procedure that yields one of a set of *outcomes*
- The set S of possible outcomes to an experiment is called the *sample space*, and a subset $E \subseteq S$ is called an *event*.
- Let S be a finite sample space. A *Probability function* is a mapping

$$P : \mathbb{P}(S) \longrightarrow \mathbb{R}$$

satisfying certain axioms (7.2).

In (7.1) we define P as

$$\boxed{P(E) = \frac{|E|}{|S|}} \quad (*)$$

for any event $E \subseteq S$.

<u>Note</u>: there are many other Probability functions. The above definition says all outcomes are equally likely.

observe

(1) for any $E \subseteq S$: $0 \leq P(E) \leq 1$

(2) $P(\phi) = 0$

(3) $P(S) = 1$

(4) If $E_1, E_2, \ldots, E_k$ are pairwise disjoint events in S, then

$$P(E_1 \cup E_2 \cup \cdots \cup E_k) = P(E_1) + P(E_2) + \cdots + P(E_k)$$

<u>Exercise</u>

Prove (1)–(4) using $*$

Ex.

A single die is thrown. What is the probability that the number on top is at least 5?

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$E = \{5, 6\}$$

$$\therefore P(E) = \frac{2}{6} = \frac{1}{3} = .3333\ldots$$

Ex.

Two dice are thrown. What is the probability that the sum of the numbers is 8?

$$\begin{array}{cccccc} 2 & 3 & 4 & 5 & 6 & 7 \end{array}$$

$$S=\left\{\begin{array}{cccccc} 11 & 12 & 13 & 14 & 15 & 16 \\ 21 & 22 & 23 & 24 & 25 & 26 \\ 31 & 32 & 33 & 34 & 35 & 36 \\ 41 & 42 & 43 & 44 & 45 & 46 \\ 51 & 52 & 53 & 54 & 55 & 56 \\ 61 & 62 & 63 & 64 & 65 & 66 \end{array}\right\}$$

sum = 8

9

10

11

12

E

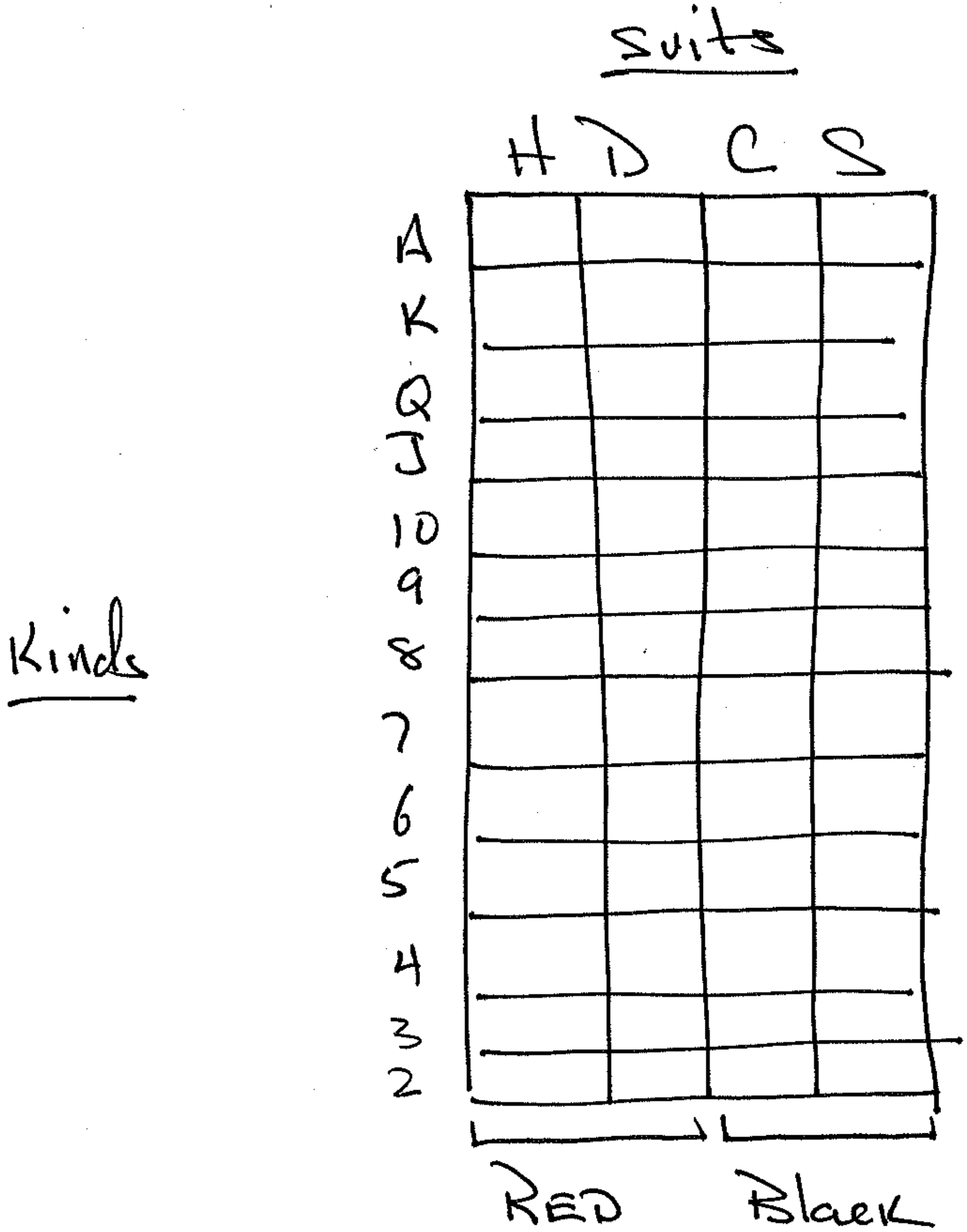
$$P(E) = \frac{5}{36} = .1388\ldots$$

also

$$P(\text{sum} = 5) = \frac{4}{36} = \frac{1}{9}$$

Ex.

What is the Probability that a 5-card Poker hand contains 4 of a Kind.



so

$S = \{$ all 5-card Poker hands $\}$

$$|S| = \binom{52}{5} = 2{,}598{,}960$$

$E = \{$ hands containing 4 of a kind $\}$

$$|E| = \binom{13}{1} \cdot \binom{4}{4} \cdot \binom{48}{1} = 624$$

$$\therefore P(E) = \frac{624}{2{,}598{,}960} = \frac{1}{4165} = .00024\ldots$$

__Ex.__ What is the probability that a 5-card Poker hand contains 3 of a kind (and not 4 of a kind)

$E = \{\text{hands containing 3 of a kind}\}$

$$|E| = \binom{13}{1}\cdot\binom{4}{3}\cdot\binom{48}{2} = 58{,}656$$

$$\therefore P(E) = \frac{58{,}656}{2{,}598{,}960}$$

$$= \frac{94}{4165} = .02256$$

Theorem

Given $E \subseteq S$, the probability of $\overline{E} = S - E$ is

$$P(\overline{E}) = 1 - P(E)$$