CSE 16 6-25-24

1.1 Propositional Logic

Defn

A proposition is a statement that can be assigned a truth value

True	or	False
T		F
1		0

Ex.
- $2+3=5$
- $2+3=7$
- 'it is raining today'

Ex. (not propositions)

- 'hello'
- $x + 2 = 7$
- $a + b = c$

we let $p, q, r \ldots$ be propositional variables.

Logical operators

- Negation : $\neg p$ (not)

p	$\neg p$
0	1
1	0

'p is false'

'not p'

Ex $p =$ 'it is raining today'

$q =$ 'today is Wednesday'

- Conjuction: $p \wedge q$ (and)

p	q	$p \wedge q$
0	0	0
0	1	0
1	0	0
1	1	1

'p and q'

'both p and q'

- Disjunction: $p \vee q$ (or, inclusive or)

p	q	$p \vee q$
0	0	0
0	1	1
1	0	1
1	1	1

'p or q'

'p or q or possibly both'

- **Exclusive or** : $p \oplus q$ (exor, or)

p	q	$p \oplus q$
0	0	0
0	1	1
1	0	1
1	1	0

'p or q, but not both'

'p or q'

Ex 'your money or your life'

- **Implication**: $p \to q$

hypothesis ↓ conclusion ↓

p	q	$p \to q$
0	0	1
0	1	1
1	0	0
1	1	1

'p implies q'

'if p, then q'

'p, only if q'

'p is sufficient for q'

also: 'q, if p'

'q, whenever p'

'q is necessary for p'

Ex. p = 'it is sunny today'

q = 'I will take you to the beach'

Related Propositions

the converse of $p \Rightarrow q$ is $q \Rightarrow p$

" contrapositive " " " $\neg q \rightarrow \neg p$

" inverse " " " $\neg p \rightarrow \neg q$

- Biconditional : $p \Leftrightarrow q$ (if and only if)

p	q	$p \Leftrightarrow q$
0	0	1
0	1	0
1	0	0
1	1	1

'p is necessary and sufficient for q'

Prop. Var. $p, q, r, \ldots$

atomic propositions

compound propositions

either atomic or built from

Prop. variables & logical ops.

Ex. $(p \to q) \to r$

p	q	r	$p \Rightarrow q$	$(p \Rightarrow q) \to r$
0	0	0	1	0
0	0	1	1	1
0	1	0	1	0
0	1	1	1	1
1	0	0	0	1
1	0	1	0	1
1	1	0	1	0
1	1	1	1	1

Ex. $p \to (q \to r)$

p	q	r	$q \to r$	$p \to (q \to r)$
0	0	0	1	1
0	0	1	1	1
0	1	0	0	1
0	1	1	1	1
1	0	0	1	1
1	0	1	1	1
1	1	0	0	0
1	1	1	1	1

Note: $p \to q \to r$ is meaningless

$$(p \to q) \to r \neq p \to (q \to r)$$

1.2 Applications

Read carefully

Translation

'unless' = 'if not'

'p unless q' = 'if not p, then q'

$= \neg p \to q$

p	q	$\neg p$	$\neg q$	$\neg p \to q$	$\neg q \to p$	$p \vee q$
0	0	1	1	0	0	0
0	1	1	0	1	1	1
1	0	0	1	1	1	1
1	1	0	0	1	1	1

$\therefore$ 'p unless q' $= \neg p \to q = p \vee q = \neg q \to p =$ 'q unless p'

1.3 Equivalences

Defn

A compound proposition that is true for every assignment of truth values to its prop. variables is called a tautology. One that is always false is called a contradiction (or unsatisfiable)

Ex.

P	$\neg P$	$P \vee \neg P$	$P \wedge \neg P$
0	1	1	0
1	0	1	0

↑ tautology ↑ contradiction

Ex. $p \Rightarrow (p \vee q)$ is a tautology

p	q	$p \vee q$	$p \Rightarrow (p \vee q)$
0	0	0	1
0	1	1	1
1	0	1	1
1	1	1	1

a Prop. that is not a contradiction is called satisfiable. A Prop. that is neither a tautology nor a contradiction is a contingency.

Ex. p

$\uparrow$

contingency, satisfiable

Defn

Two compound Propositions A, B are said to be **logically equivallent** iff $A \leftrightarrow B$ is a tautology.

Notation $A \equiv B$, $A \Leftrightarrow B$

Ex. $p \equiv p$

show $p \Leftrightarrow p$ is a tautology

p	$p \leftrightarrow p$
0	1
1	1

Ex. $p \to q \equiv \neg p \vee q$

Ex. $p \to q \overset{\checkmark}{\equiv} \neg q \to \neg p$ (contrapositive)

Ex. $p \to q \not\equiv q \to p$ (converse)

p	q	$\neg p$	$\neg q$	$p \to q$	$\neg q \to \neg p$	$q \to p$
0	0	1	1	1	1	1
0	1	1	0	1	1	0
1	0	0	1	0	0	1
1	1	0	0	1	1	1

same

different