cse 16 7-7-23

- Quiz 1 today: 12:15 – 12:45
- hw1 solns posted.

Defn
The <u>cartesian product</u> of A, B is

$$A \times B = \{(x,y) \mid x \in A \wedge y \in B\}$$

<u>Ex.</u> $A = \{1,2,3\}$, $B = \{3,4\}$

$$A \times B = \{(1,3),(1,4),(2,3),(2,4),(3,3),(3,4)\}$$

$$B \times A = \{(3,1),(4,1),(3,2),(4,2),(3,3),(4,3)\}$$

Theorem

If A and B are finite, then so is $A \times B$, and

$$|A \times B| = |A| \cdot |B| .$$

more generally: if $A_1, A_2, \ldots, A_n$ then

$$A_1 \times A_2 \times \cdots \times A_n = \{(x_1, x_2, \ldots, x_n) \mid x_i \in A_i \text{ for } 1 \leq i \leq n\}$$

and

$$|A_1 \times A_2 \times \cdots \times A_n| = |A_1| \cdot |A_2| \cdots |A_n| .$$

Russell's Paradox (P.146 #46)

Let $\mathcal{S} = \{\text{all sets}\}$. observe $\mathcal{S} \in \mathcal{S}$ but $\emptyset \notin \emptyset$. now define

$$\mathcal{R} = \{A \in \mathcal{S} \mid A \notin A\}$$

Either $\mathcal{R} \in \mathcal{R}$ or $\mathcal{R} \notin \mathcal{R}$. But

$$\mathcal{R} \in \mathcal{R} \rightarrow \mathcal{R} \notin \mathcal{R} \quad \cdot\!\times\!\cdot$$

and

$$\mathcal{R} \notin \mathcal{R} \rightarrow \mathcal{R} \in \mathcal{R} \quad \cdot\!\times\!\cdot$$

There are other Paradoxical sets:

<u>Ex</u> $A = \{B\}$, $B = \{A\}$.

<u>Solution</u> Axiomatic set theory.

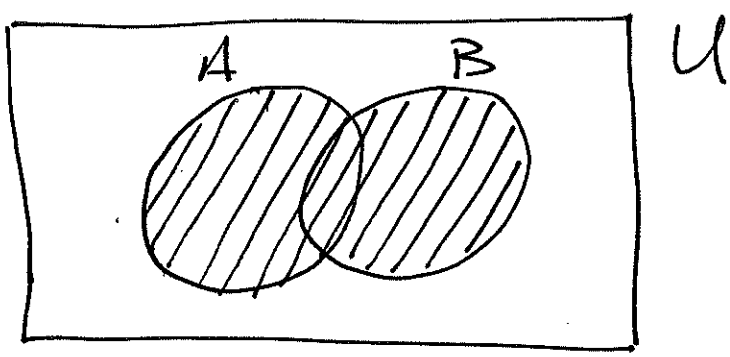
2.2 Set operations

Defn

The union of A, B (where $A \subseteq U$, $B \subseteq U$.)

$$A \cup B = \{x \in U \mid x \in A \vee x \in B\}$$

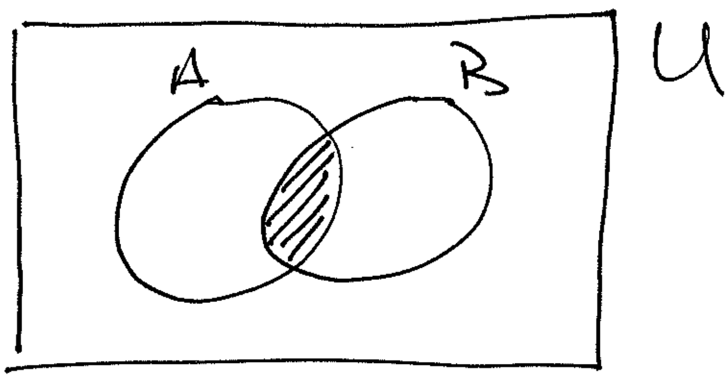
Venn diagram:



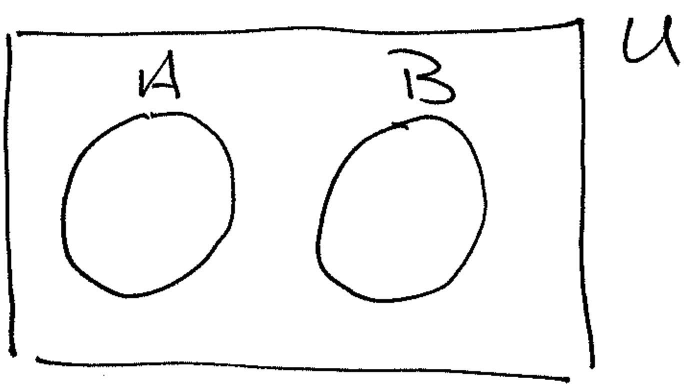
Defn

The intersection of A, B is

$$A \cap B = \{x \in U \mid x \in A \wedge x \in B\}$$



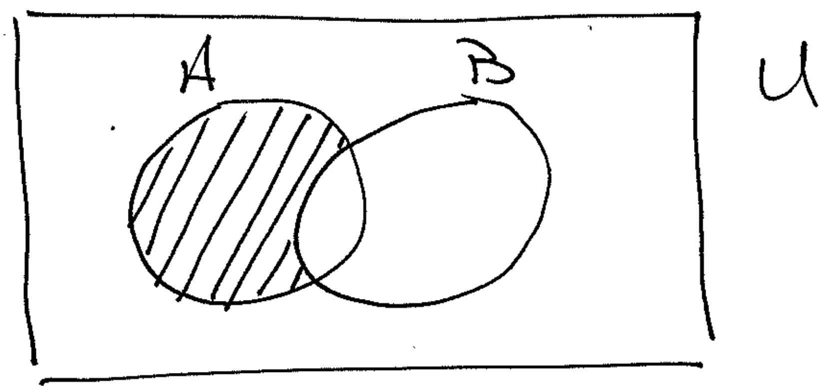
We say A and B are disjoint iff $A \cap B = \phi$.



<u>Defn</u>

The <u>set difference</u> of A with B

$$A - B = \{x \in U \mid x \in A \wedge x \notin B\}$$

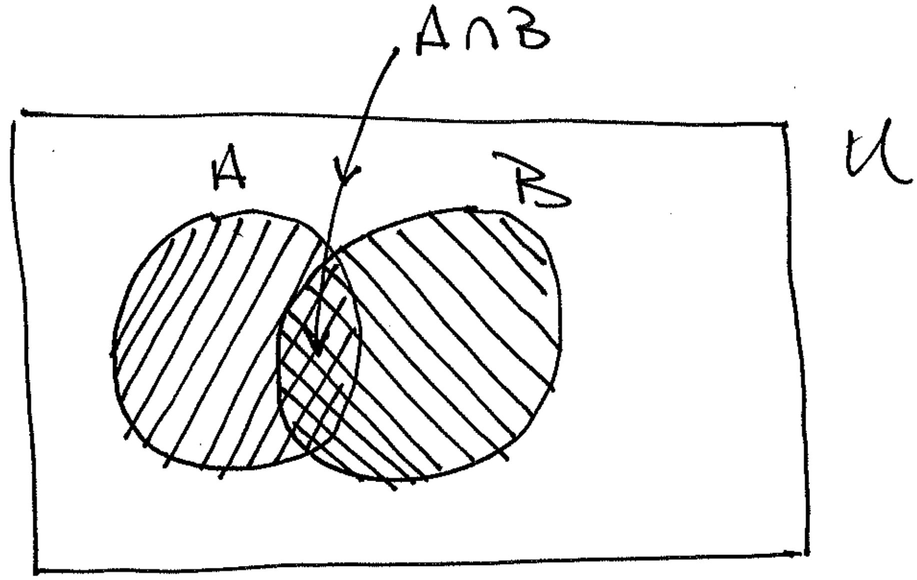


<u>Theorem</u>

If A, B are finite, then so are $A \cup B$ and $A \cap B$, and

$$|A \cup B| = |A| + |B| - |A \cap B|$$

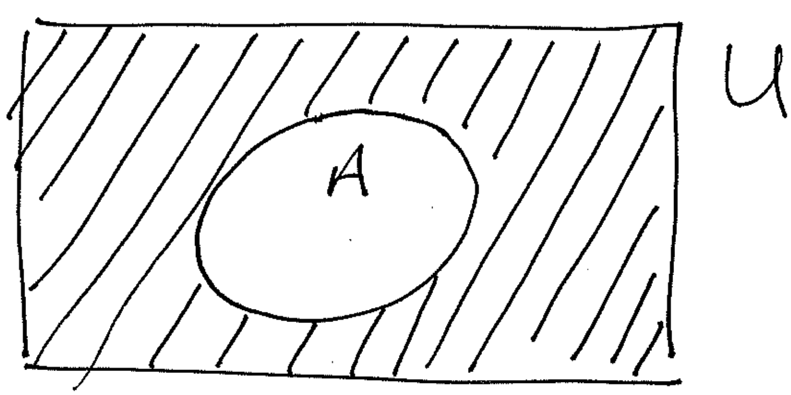
(sp. case of Principle of inclusion-exclusion (PIE).)



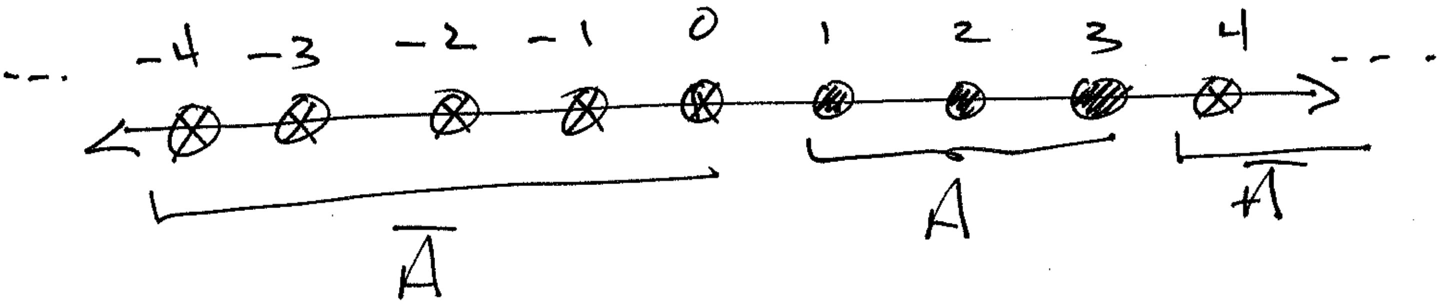
<u>Defn</u>

The <u>complement</u> of A is the set

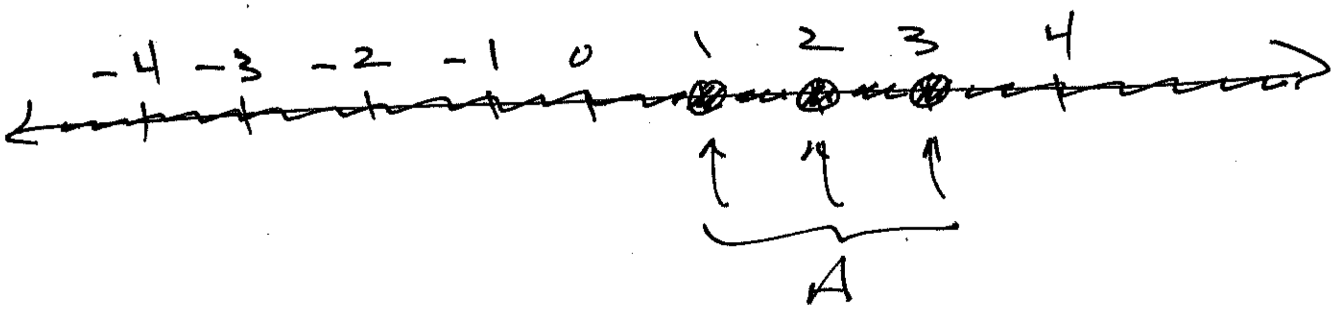
$$\bar{A} = U - A = \{x \in U \mid x \notin A\}$$



Ex. $\mathcal{U} = \mathbb{Z}$, $A = \{1, 2, 3\}$



Ex. $\mathcal{U} = \mathbb{R}$, $A = \{1, 2, 3\}$



$$\overline{A} = \{x \in \mathbb{R} \mid x \neq 1 \wedge x \neq 2 \wedge x \neq 3\}$$

Set identities (table 1 p.130)

- Identity $A \cap U = A$

  $A \cup \emptyset = A$

- Domination $A \cup U = U$

  $A \cap \emptyset = \emptyset$

- Idempotent $A \cup A = A$

  $A \cap A = A$

- Double complement $\overline{\overline{A}} = A$

- commutative $A \cup B = B \cup A$

  $A \cap B = B \cap A$

- Associative $(A \cup B) \cup C = A \cup (B \cup C)$

  $\cap \quad \cap \qquad \cap \quad \cap$

- Distributive $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

  $\cap \quad \cup \qquad \cap \quad \cup \quad \cap$

- DeMorgan $\overline{A \cup B} = \bar{A} \cap \bar{B}$

  $\overline{A \cap B} = \bar{A} \cup \bar{B}$

- Absorption $A \cup (A \cap B) = A$

  $A \cap (A \cup B) = A$

- Complement $A \cup \bar{A} = U$

  $A \cap \bar{A} = \phi$

Prove 1st Demorgan: logical identites.

$$\overline{A \cup B} = \{x \in U \mid x \notin A \cup B\}$$

$$= \{x \in U \mid \neg (x \in A \cup B)\}$$

$$= \{x \mid \neg (x \in A \lor x \in B)\}$$

$$= \{x \mid \neg x \in A \land \neg x \in B\}$$

$$= \{x \mid x \in \bar{A} \land x \in \bar{B}\}$$

$$= \bar{A} \cap \bar{B}$$

Prove 1st Distributive : Membership table

1 = membership

0 = non-membership

A	B	C	$B \cap C$	$A \cup (B \cap C)$	$A \cup B$	$A \cup C$	$(A \cup B) \cap (A \cup C)$
0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0
0	1	0	0	0	1	0	0
0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	1	0	0	1	1	1	1
1	1	1	1	1	1	1	1

Venn diag for dist. law