cse 16 6-28-23

(1.1) Propositional logic

Defn

A Proposition is a statement that can be assigned a truth value:

True or False

T F

1 0

Ex Propositions

- 'it is raining today'
- '$2+3=5$'
- '$2+3=7$'

Ex. not propositions

- 'hello'
- '$x + 2 = 7$'
- '$a + b = c$'

Propositional variables: $p, q, r, s, \ldots$

Logical operators

Negation: $\neg p$ (not)

p	$\neg p$
0	1
1	0

'p is false'

'not p'

'it is not the case that p'

running examples

p = 'it is raining today'

q = 'today is wednesday'

<u>Conjunction</u>: $p \wedge q$ (and)

p	q	$p \wedge q$
0	0	0
0	1	0
1	0	0
1	1	1

'p and q'

'both p and q are true'.

Disjunction: $p \vee q$ (or, inclusive or)

p	q	$p \vee q$
0	0	0
0	1	1
1	0	1
1	1	1

'p or q'

'p or q, or possibly both'

Exclusive or: $p \oplus q$ (exor, or)

p	q	$p \oplus q$
0	0	0
0	1	1
1	0	1
1	1	0

'p or q, but not both'

Ex. 'your money or your life'

Implication: $p \to q$

p	q	$p \to q$
0	0	1
0	1	1
1	0	0
1	1	1

'p implies q'
'if p, then q'
'p, only if q'
'p is sufficient for q'

also:

'q, if p'
'q, whenever p'
'q is necessary for p'

in $p \to q$ we call

p: hypothesis
q: conclusion

Ex. p = 'it is sunny today', q = 'I will take you to the beach'

Propositions related to $p \to q$:

- <u>converse</u> of $p \to q$: $q \to p$
- <u>contrapositive</u> " ": $\neg q \to \neg p$
- <u>inverse</u> " " : $\neg p \to \neg q$

<u>Biconditional</u>: $p \leftrightarrow q$ (if and only if)

p	q	$p \leftrightarrow q$
0	0	1
0	1	0
1	0	0
1	1	1

'p is necessary and sufficient for q'

<u>Atomic Propositions</u> : $p, q, r, \ldots$

<u>compound Propositions</u> : Propositions Built from atomic Prop. using logical operators.

<u>Ex.</u> $(p \rightarrow q) \rightarrow r$

p	q	r	$p \rightarrow q$	$(p \rightarrow q) \rightarrow r$
0	0	0	1	0
0	0	1	1	1
0	1	0	1	0
0	1	1	1	1
1	0	0	0	1
1	0	1	0	1
1	1	0	1	0
1	1	1	1	1

Ex. $p \to (q \to r)$

p	q	r	$q \to r$	$p \to (q \to r)$
0	0	0	1	1
0	0	1	1	1
0	1	0	0	1
0	1	1	1	1
1	0	0	1	1
1	0	1	1	1
1	1	0	0	0
1	1	1	1	1

Thus: $(p \to q) \to r \neq p \to (q \to r)$

and the expression

$$p \to q \to r$$

is meaningless

Exercise: show that

$$p \vee q \wedge r$$

is meaningless, i.e.

$$(p \vee q) \wedge r \neq p \vee (q \wedge r)$$

since they have different truth tables.

1.2 Applications

Translation (read this)

'unless' = 'if not'

i.e. 'p unless q' = 'if not p then q'

$= \neg p \rightarrow q$

p	q	$\neg p$	$\neg q$	$\neg p \rightarrow q$	$\neg q \rightarrow p$	$p \vee q$
0	0	1	1	0	0	0
0	1	1	0	1	1	1
1	0	0	1	1	1	1
1	1	0	0	1	1	1

so $\neg p \rightarrow q = \neg q \rightarrow p = p \vee q$

Read also in 1.2 :

- system specifications
- Logic Puzzles
- Logic circuits

1.3 Equivalences

Defn

tautology: true for every assignment of truth values to its prop. vars.

contradiction: false " " "
" " " " " " " .

p	$\neg p$	$p \vee \neg p$	$p \wedge \neg p$
0	1	1	0
1	0	1	0

$\uparrow$ tautology $\qquad$ $\uparrow$ contradiction

Ex. $p \to (p \vee q)$ is a tautology ✓

p	q	$p \vee q$	$p \to (p \vee q)$
0	0	0	1
0	1	1	1
1	0	1	1
1	1	1	1

<u>contingency</u>: neither a tautology, nor a contradiction.

<u>satisfiable</u>: not a contradiction

Ex. p