

CSE 16
Summer 2022
Quiz 1

Solutions

1. (20 Points) In a group of 35 people, all of whom drink coffee or tea (or both), 18 drink coffee and 12 drink both coffee and tea.

- a. (10 Points) How many in the group drink tea?

Solution:

Let $A = \{\text{coffee drinkers}\}$ and $B = \{\text{tea drinkers}\}$. By the principle of inclusion-exclusion, we have $|A \cup B| = |A| + |B| - |A \cap B|$. The given information says $|A \cup B| = 35$, $|A| = 18$ and $|A \cap B| = 12$. Therefore $35 = 18 + |B| - 12$, whence $|B| = 29$. Thus **29 people drink tea.** ■

- b. (10 Points) How many drink only tea (i.e. tea but not coffee)?

Solution:

Using the same notation as in (a), we have $|B - A| = |B - (A \cap B)| = 29 - 12 = 17$. Therefore **17 people drink only tea.** ■

2. (20 Points) Let $S = \{1, 2, 3, 4\}$. Determine the set $\{A \in \mathbb{P}(S) : |A| = 2\}$.

Solution:

Observe that $\{A \in \mathbb{P}(S) : |A| = 2\} = \{A \subseteq S : |A| = 2\}$. Therefore we seek the set of all 2-element subsets of S , and hence

$$\{A \in \mathbb{P}(S) : |A| = 2\} = \{\{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}\}.$$

■

3. (10 Points) Determine whether the logical expression $(p \oplus q) \vee (q \Rightarrow \sim r)$ is a tautology. Either prove that it is a tautology (by any method), or show that it is not by finding an assignment of truth values to the propositional variables p, q, r that makes the expression false. Justify your answer and the reasoning that led to it.

Solution:

We construct a truth table for the expression.

p	q	r	$p \oplus q$	$\sim r$	$q \Rightarrow \sim r$	$(p \oplus q) \vee (q \Rightarrow \sim r)$
0	0	0	0	1	1	1
0	0	1	0	0	1	1
0	1	0	1	1	1	1
0	1	1	1	0	0	1
1	0	0	1	1	1	1
1	0	1	1	0	1	1
1	1	0	0	1	1	1
1	1	1	0	0	0	0

We conclude that $(p \oplus q) \vee (q \Rightarrow \sim r)$ is **not a tautology**, since the assignment $p = q = r = T$ makes it false.