CSE 16
Homework Assignment 2
Solutions to Selected Problems

(2.5) 10

Suppose the statement $\big((P \wedge Q) \vee R\big) \Rightarrow (R \vee S)$ is false. Find the truth values of P, Q, R and S. (Note this can be done without constructing a truth table.)

Solution:

The only way for an implication to be false is for the hypothesis to be true while the conclusion is false. Thus $(P \wedge Q) \vee R$ is true, and $R \vee S$ is false. Since $R \vee S$ is false, both operands must be false, i.e.

$$R = S = F.$$

Since R is false and $(P \wedge Q) \vee R$ is true, we must have $P \wedge Q$ true, hence both operands are true, i.e.

$$P = Q = T.$$

Thus if $\big((P \wedge Q) \vee R\big) \Rightarrow (R \vee S)$ is false, then $\boldsymbol{P = Q = T}$**, and** $\boldsymbol{R = S = F}$. ■

(2.6) 10

Decide whether $(P \Rightarrow Q) \vee R$ and $\sim \big((P \wedge \sim Q) \wedge \sim R\big)$ are logically equivalent.

Solution:

P	Q	R	$P \Rightarrow Q$	$(P \Rightarrow Q) \vee R$	$\sim Q$	$P \wedge \sim Q$	$\sim R$	$(P \wedge \sim Q) \wedge \sim R$	$\sim \big((P \wedge \sim Q) \wedge \sim R\big)$
0	0	0	1	1	1	0	1	0	1
0	0	1	1	1	1	0	0	0	1
0	1	0	1	1	0	0	1	0	1
0	1	1	1	1	0	0	0	0	1
1	0	0	0	0	1	1	1	1	0
1	0	1	0	1	1	1	0	0	1
1	1	0	1	1	0	0	1	0	1
1	1	1	1	1	0	0	0	0	1

Since the two expressions have the same truth table, they are **logically equivalent**. ■

Alternate Solution:

Using logical identities, we have

$$\begin{aligned}
\sim \big((P \wedge \sim Q) \wedge \sim R\big) &= \sim(P \wedge \sim Q) \vee \sim(\sim R) && \text{DeMorgan} \\
&= \big(\sim P \vee \sim(\sim Q)\big) \vee R && \text{DeMorgan \& Double Negation} \\
&= (\sim P \vee Q) \vee R && \text{Double Negation} \\
&= (P \Rightarrow Q) \vee R && \text{Implication}
\end{aligned}$$

It follows that $(P \Rightarrow Q) \vee R = \sim \big((P \wedge \sim Q) \wedge \sim R\big)$, so the expressions are **logically equivalent**. ■

(2.6) 12

Decide whether $\sim(P \Rightarrow Q)$ and $P \wedge \sim Q$ are logically equivalent.

Solution:

P	Q	$P \Rightarrow Q$	$\sim(P \Rightarrow Q)$	$\sim Q$	$P \wedge \sim Q$
0	0	1	0	1	0
0	1	1	0	0	0
1	0	0	1	1	1
1	1	1	0	0	0

Since the two expressions have the same truth table, they are **logically equivalent**. ■

Alternate Solution:

Using logical identities, we have

$$\begin{aligned} \sim(P \Rightarrow Q) &= \sim(\sim P \vee Q) && \text{Implication} \\ &= \sim\sim P \wedge \sim Q && \text{DeMorgan} \\ &= P \wedge \sim Q && \text{Double Negation} \end{aligned}$$

It follows that the two expressions are **logically equivalent**. ■

(2.7) 10

Translate $\exists m \in \mathbb{Z}, \forall n \in \mathbb{Z}, m = n + 5$ into natural language, and determine whether it is true or false.

Solution:

Either one of the following translations work.

"There exists an integer m such that for all integers n, we have $m = n + 5$"

or

"There is an integer that is 5 more than every integer"

This statement is **false**. ■