CSE 16
Homework Assignment 1
Solutions to Selected Problems

(1.2) 2g
Let $A = \{\pi, e, 0\}$ and $B = \{0, 1\}$. Then

$$B \times B = \{\ (0,0), (0,1), (1,0), (1,1)\ \}$$

and therefore

$$\boldsymbol{A \times (B \times B) = \{\ \big(\pi, (0,0)\big), \big(\pi, (0,1)\big), \big(\pi, (1,0)\big), \big(\pi, (1,1)\big), \big(e, (0,0)\big), \big(e, (0,1)\big), \big(e, (1,0)\big), \big(e, (1,1)\big), \big(0, (0,0)\big), \big(0, (0,1)\big), \big(0, (1,0)\big), (0, (1,1))\ \}}$$

(1.3) 8
Let $S = \big\{\ \{0,1\}, \{0,1,\{2\}\}, \{0\}\ \big\}$. Then

$$\mathbb{P}(S) = \Big\{\ \emptyset, \{\{0,1\}\}, \big\{\{0,1,\{2\}\}\big\}, \{\{0\}\}, \big\{\{0,1\}, \{0,1,\{2\}\}\big\}, \{\{0,1\}, \{0\}\}, \big\{\{0,1,\{2\}\}, \{0\}\big\}, \{\ \{0,1\}, \{0,1,\{2\}\}, \{0\}\ \}\ \Big\}$$

<u>Note to graders:</u> the outer braces are not necessary for full credit, since the problem said "list all subsets", not "determine the power set".

(1.4) 20
Let $|A| = m$. Then $|\mathbb{P}(A)| = 2^m$, and therefore the number of 1-element subsets of $\mathbb{P}(A)$ is also 2^m. There is only one 0-element subset of $\mathbb{P}(A)$, namely $\emptyset$. Thus

$$\{\, X \subseteq \mathbb{P}(A) : |X| \leq 1 \,\} = \{\, X \subseteq \mathbb{P}(A) : |X| = 1 \,\} \cup \{\, X \subseteq \mathbb{P}(A) : |X| = 0 \,\}$$

and hence

$$|\{\, X \subseteq \mathbb{P}(A) : |X| \leq 1 \,\}| = |\{\, X \subseteq \mathbb{P}(A) : |X| = 1 \,\}| + |\{\, X \subseteq \mathbb{P}(A) : |X| = 0 \,\}|$$

$$= \boldsymbol{2^m + 1}$$

(1.8) 14
For each $\alpha \in I$, let A_α be a set. Suppose that $\emptyset \neq J \subseteq I$. It follows that

$$\bigcap_{\alpha \in I} A_\alpha \subseteq \bigcap_{\alpha \in J} A_\alpha$$

Solution:
Let x be an element of the left hand side (LHS). Then $x \in A_\alpha$ for all $\alpha \in I$. Since $J \subseteq I$, this implies $x \in A_\alpha$ for all $\alpha \in J$. Therefore x is also a member of the right hand side (RHS), and hence LHS $\subseteq$ RHS. ■