CSE 16 8-11-22

- Final today

<u>Review</u>

<u>11.4 #6</u> P partition of $\mathbb{Z}$.

$$P = \{\{0\}, \{-1, 1\}, \{-2, 2\}, \dots\}$$

eq. relation:

$$R = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} \mid |x| = |y|\}$$

$$x = \pm y$$

$$x^2 = y^2$$

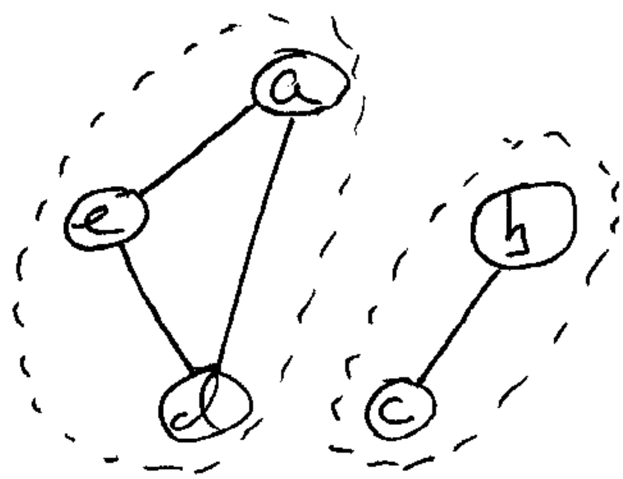
i.e. xRy iff

11.3 #2

Let $A = \{a, b, c, d, e\}$, let $R \subseteq A \times A$ be an eq. rel. Suppose:

aRd, bRc, eRd and there are 2 equiv. classes.

Determine R.



$$R = \{ (a,a), (b,b), (c,c), (d,d), (e,e), (a,d), (d,a), (a,e), (e,a), (d,e), (e,d), (b,c), (c,b) \}$$

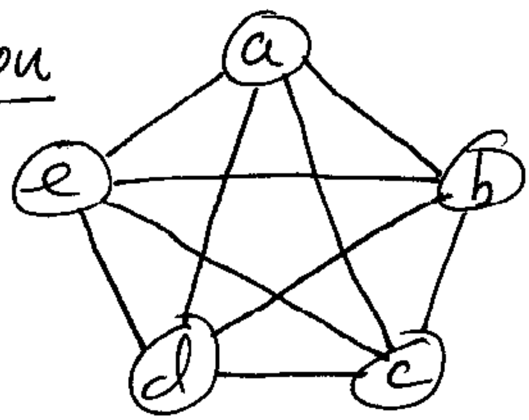
$$P = \{ \{a,d,e\}, \{b,c\} \}.$$

11.3 #4

$A = \{a, b, c, d, e\}$ $R \subseteq A \times A$ an eq. rel. Assume: aRd, bRc, eRa, cRe.

Determine R:

full relation



1 eq. class

$$R = A \times A$$

12.2 #2

$\ln : (0, \infty) \longrightarrow \mathbb{R}$

defn

$\boxed{\ln(x) = y}$

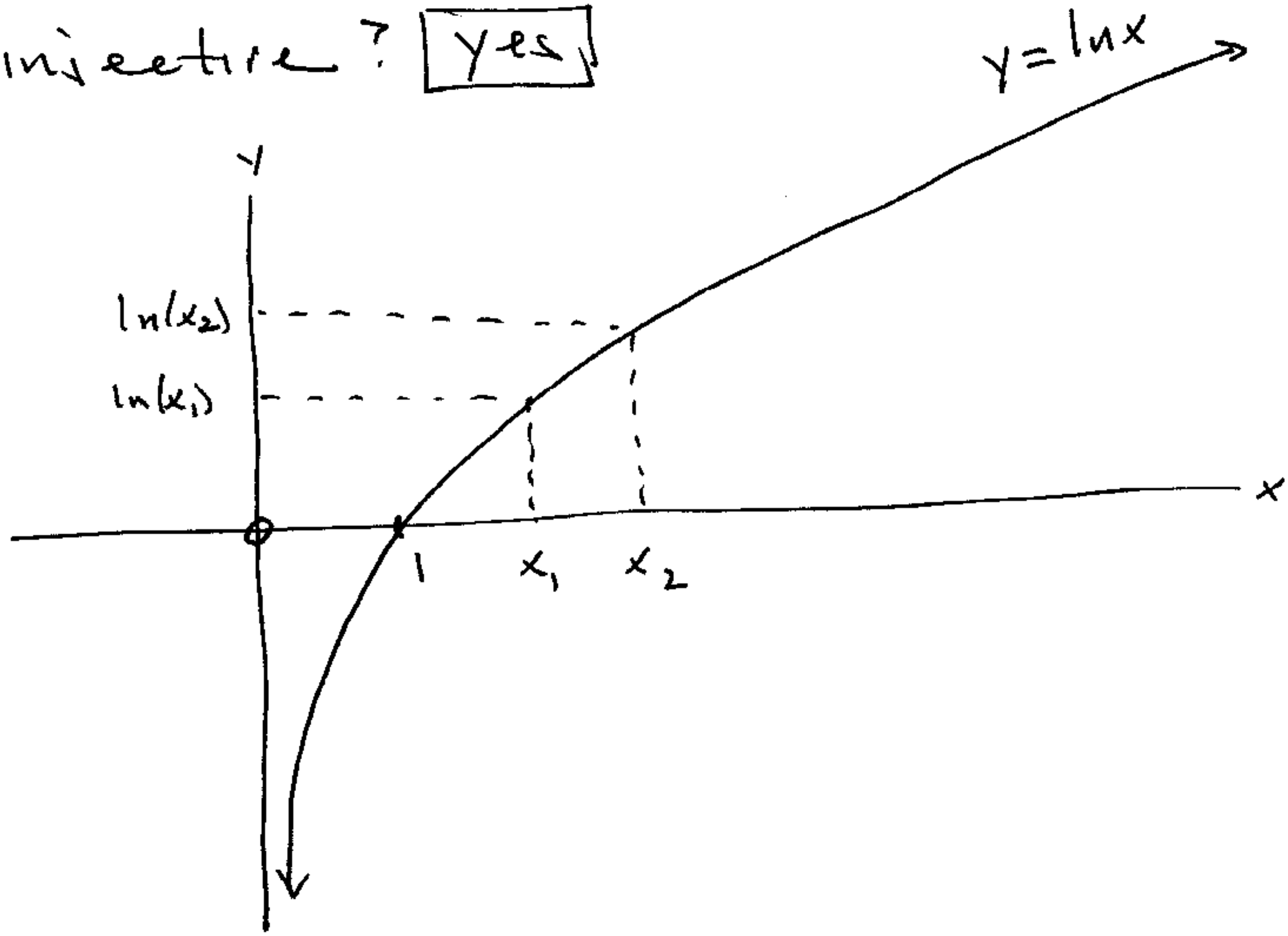
$\Updownarrow$

$\boxed{e^y = x}$

injective? $\boxed{\text{yes}}$

surjective? $\boxed{\text{yes}}$

bijective? $\boxed{\text{yes}}$



11.5 #2

$+$ and $\cdot$ for $\mathbb{Z}_3$

+	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

$(3k_1)+(3k_2+2)$
$= 3(k_1+k_2)+2$

$(3k_1+2)+(3k_2+2)$
$= 3(k_1+k_2)+3+1$
$= 3(k_1+k_2+1)+1$

or

$[2]+[2]=[2+2]$
$=[4]=[1]$

$\cdot$	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

11.5 #6

if $[a], [b] \in \mathbb{Z}_6$,
and $[a]\cdot[b] = 0$, does it follow that
$[a] = 0$ or $[b] = 0$. ans: $\boxed{\text{NO}}$

$\cdot$	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	0	2	4
3	0	3	0	3	0	3
4	0	4	2	0	4	2
5	0	5	4	3	2	1

what if $[a], [b] \in \mathbb{Z}_7$?

ans. $\boxed{\text{yes}}$

ch 10 #29

$$\forall n \geq 0 : \boxed{\sum_{k=0}^{n} \binom{n-k}{k} = F_{n+1}} \leftarrow P(n)$$

convention: $\binom{a}{b} = 0$ if $a<b$ or $b<0$

i.e. if not: $0 \leq b \leq a$.

Proof (II d with 2 base cases)

I. $n=0$: $\binom{0}{0} = 1 = F_1 = F_{0+1}$ ✓

$n=1$: $\binom{1}{0} + \cancel{\binom{0}{1}} = \binom{1}{0} = 1 = F_2 = F_{1+1}$ ✓

II d. $\forall n>1 : P(0) \wedge P(1) \wedge \dots \wedge P(n-1) \Rightarrow P(n)$

let $n>1$ be arbitrary. Assume for all l in range $0 \leq l < n$ that

$$\sum_{k=0}^{l} \binom{l-k}{k} = F_{l+1} .$$

we must show

$$\sum_{k=0}^{n} \binom{n-k}{k} = F_{n+1} \ .$$

Pascal

so

$$\sum_{k=0}^{n} \binom{n-k}{k} = \sum_{k=0}^{n} \left[\binom{n-k-1}{k-1} + \binom{n-k-1}{k} \right]$$

$$= \sum_{k=0}^{n} \binom{n-k-1}{k-1} + \sum_{k=0}^{n} \binom{n-k-1}{k}$$

replace k by k+1

$$= \sum_{k=-1}^{n-1} \binom{n-(k+1)-1}{k} + \sum_{k=0}^{n} \binom{n-k-1}{k}$$

$$= \sum_{k=-1}^{n-1} \binom{(n-2)-k}{k} + \sum_{k=0}^{n} \binom{(n-1)-k}{k}$$

$$= \cancel{\binom{n-1}{-1}} + \sum_{k=0}^{n-2} \binom{(n-2)-k}{k} + \cancel{\binom{-1}{n-1}} + \sum_{k=0}^{n-1} \binom{(n-1)-k}{k} + \cancel{\binom{-1}{n}}$$

$$= F_{(n-2)+1} + F_{(n-1)+1} \quad \left\{ \begin{array}{l} \text{by ind.} \\ \text{hyp. with } \ell = n-2 \\ \text{and } \ell = n-1 \end{array} \right.$$

$$= F_{n-1} + F_n$$

$$= F_{n+1} \qquad . \qquad \blacksquare$$