cse 16 6-28-22

(2.3) conditional operation

continue ...

<u>Ex.</u> p = 'it is sunny today'

q = 'I will take you to the beach'

$p \to q$ = 'if it is sunny today, then I will take you to the beach'

<u>note</u>: $F \to T$ is sometimes called <u>vacuously true</u>.

Related Propositions to $p \to q$:

- Converse : $q \to p$
- Contrapositive : $\sim q \to \sim p$
- Inverse : $\sim p \to \sim q$

note: $\sim$ has higher precedence than $\to$ (and all other ops), so

$$\sim p \to q \text{ means } (\sim p) \to q$$

and $\sim p \vee q$ means $(\sim p) \vee q$

(2.4) biconditional operator

Biconditional : $p \Leftrightarrow q$ (also $p \leftrightarrow q$)

p	q	$p \Leftrightarrow q$
F	F	T
F	T	F
T	F	F
T	T	T

true only when the operands the same value, false otherwise.

<u>read as</u>:

'p if and only if q'

'p is necessary and sufficient for q'

'p is equivalent to q'.

(2.5) Truth tables

- <u>atomic statements</u>: p, q, r, s...

  (i.e. Propositional variables)

- <u>compound statements</u>:

  - atomic statements, or
  - built from atomic statements using logical operators.

Ex. $(p \leftrightarrow q) \vee (\sim q \rightarrow r)$

p	q	r	$p \leftrightarrow q$	$\sim q$	$\sim q \rightarrow r$	$(p \leftrightarrow q) \vee (\sim q \rightarrow r)$
0	0	0	1	1	0	1
0	0	1	1	1	1	1
0	1	0	0	0	1	1
0	1	1	0	0	1	1
1	0	0	0	1	0	0
1	0	1	0	1	1	1
1	1	0	1	0	1	1
1	1	1	1	0	1	1

$0 = F$
$1 = T$

Ex. $\sim (p \vee q)$

p	q	$p \vee q$	$\sim(p \vee q)$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

Ex. $p \to (q \to r)$

p	q	r	$q \to r$	$p \to (q \to r)$
0	0	0	1	1
0	0	1	1	1
0	1	0	0	1
0	1	1	1	1
1	0	0	1	1
1	0	1	1	1
1	1	0	0	0
1	1	1	1	1

Ex. $(p \to q) \to r$

p	q	r	$p \to q$	$(p \to q) \to r$
0	0	0	1	0
0	0	1	1	1
0	1	0	1	0
0	1	1	1	1
1	0	0	0	1
1	0	1	0	1
1	1	0	1	0
1	1	1	1	1

$\therefore$ $p \to q \to r$ makes no sense!!

Exercise

do both

- $(p \vee q) \wedge r$
- $p \vee (q \wedge r)$

observe $p \vee q \wedge r$ makes no sense !!

(2.6) logical equivalences

Defn

A tautology is a compound stmt. that is true for all assignments of truth values to its prop. variables.

Ex. $p \vee (\sim p)$

p	$\sim p$	$p \vee \sim p$
0	1	1
1	0	1

Ex. $p \rightarrow p$

p	$p \rightarrow p$
0	1
1	1

Ex. $p \rightarrow (p \vee q)$

p	q	$p \vee q$	$p \rightarrow (p \vee q)$
0	0	0	1
0	1	1	1
1	0	1	1
1	1	1	1

Defn

A contradiction is a compound stmt that is false for all assignments of truth values to its propositional variables.

Ex. $p \wedge (\sim p)$

p	$\sim p$	$p \wedge (\sim p)$
0	1	0
1	0	0

Defn

a statement that is not a contradiction is said to be <u>satisfiable</u>.
(i.e. at least one row is true)

Ex. p

p
0
1

Defn

Two compound stmts. X and Y are said to be logically equivalent if the stmt

$$X \Leftrightarrow Y$$

is a tautology. In this case we write

$$X = Y.$$

Rmk.

equivalent defn: X and Y have same truth table.

Rmk.

This is what is meant by $=$ in algebra.

for instance

$$a(b+c) = ab + ac$$

means that the number on LHS is same as number on RHS for any assignment of numerical values for a, b, c.

Ex. $p \Rightarrow q \;=\; \sim p \vee q$ (implication law)

p	q	$p \Rightarrow q$	$\sim p$	$\sim p \vee q$	$(p \Rightarrow q) \leftrightarrow (\sim p \vee q)$
0	0	1	1	1	1
0	1	1	1	1	1
1	0	0	0	0	1
1	1	1	0	1	1

same

Ex $p \to q \equiv \sim q \to \sim p$ (contrapositive law)

p	q	$p \to q$	$\sim q$	$\sim p$	$\sim q \to \sim p$
0	0	1	1	1	1
0	1	1	0	1	1
1	0	0	1	0	0
1	1	1	0	0	1

same

Ex. $p \to q \not\equiv q \to p$

p	q	$p \to q$	$q \to p$
0	0	1	1
0	1	1	0
1	0	0	1
1	1	1	1

not same!

Exercise: Prove all identities on P. 52 of BOP.

Contrapositive law

$$P \Rightarrow Q \;=\; \sim Q \Rightarrow \sim P$$

DeMorgan's laws

$$\sim(P \wedge Q) \;=\; \sim P \vee \sim Q$$
$$\sim(P \vee Q) \;=\; \sim P \wedge \sim Q$$

Commutative laws

$$P \wedge Q \;=\; Q \wedge P$$
$$P \vee Q \;=\; Q \vee P$$

Distributive laws

$$P \wedge (Q \vee R) = (P \wedge Q) \vee (P \wedge R)$$
$$P \vee (Q \wedge R) = (P \vee Q) \wedge (P \vee R)$$

Associative laws

$$P \wedge (Q \wedge R) = (P \wedge Q) \wedge R$$
$$P \vee (Q \vee R) = (P \vee Q) \vee R$$

Implication Law

$$P \Rightarrow Q \;=\; \sim P \vee Q$$

Pf. of 1st DeMorgan

$$\sim(p \wedge q) = \sim p \vee \sim q \quad \checkmark$$

p	q	$p \wedge q$	$\sim(p \wedge q)$	$\sim p$	$\sim q$	$\sim p \vee \sim q$
0	0	0	1	1	1	1
0	1	0	1	1	0	1
1	0	0	1	0	1	1
1	1	1	0	0	0	0

same

we can use these identities to prove other identities 'algebraically'

<u>EX.</u> $(p \vee q) \to r = (p \to r) \wedge (q \to r) \quad \checkmark$

<u>Proof:</u>

$$(p \vee q) \to r = \sim(p \vee q) \vee r \quad \text{(implication)}$$
$$= (\sim p \wedge \sim q) \vee r \quad \text{(deMorg.)}$$

$= (\sim p \vee r) \wedge (\sim q \vee r)$ (dist. & comm.)

$= (p \Rightarrow r) \wedge (q \Rightarrow r)$ (implication)

more identities .....

Domination

$p \vee T = T$

$p \wedge F = F$

⋮