CSE 16 6-21-22

Chap. 1 Sets

Defn (Naive)
A set is any unordered collection of objects of any kind.

The objects belonging to a set S are called members or elements.

notation: $x \in S$

To specify a set, can list its elements

Ex $\{1, 2, 3\}$

$\{1, 2, 3, \ldots, 99, 100\}$

$\{1, 2, 3, \ldots\}$

important sets

- Natural Numbers

$$\mathbb{N} = \{1, 2, 3, \ldots\}$$

- integers

$$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$$

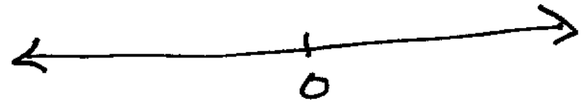
$$\mathbb{Z}^+ = \{1, 2, 3, \ldots\}$$

$$\mathbb{Z}^- = \{-1, -2, -3, \ldots\}$$

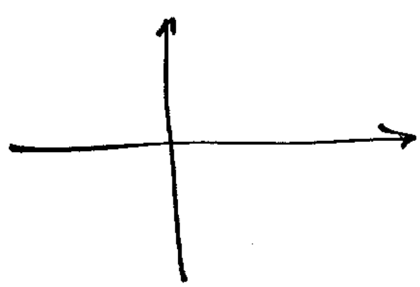
- rational numbers:

$$\mathbb{Q} = \{\text{fractions}\}$$

- Real numbers: $\mathbb{R}$



- Complex numbers: $\mathbb{C}$



- empty set

$\phi = \{ \}$ no elements

Two sets are equal if and only if they have same members.

$\{1, 2, 3\} = \{1, 3, 2\} = \{2, 1, 3\} = \cdots$

$= \{1, 2, 1, 1, 2, 3, 3, 2, 1\}$

<u>Set builder notation</u>

$A = \{ \text{expression} \mid \text{property} \}$

A is all values of <u>expression</u> satisfying <u>property</u>.

Ex $\{x \mid x \text{ is a prime number}\}$

$= \{2, 3, 5, 7, 11, 13, 17, 23, \ldots\}$

$= \{n \in \mathbb{N} \mid n \text{ is prime}\}$

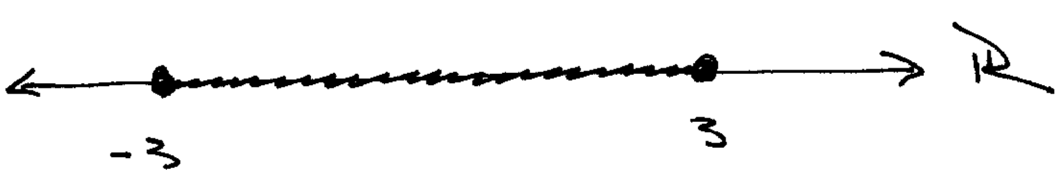
Ex. $\mathbb{Q} = \left\{\frac{p}{q} \;\middle|\; p, q \in \mathbb{Z} \text{ and } q \neq 0\right\}$

Ex $\{n \in \mathbb{N} \mid |n| \leq 3\}$

$= \{-3, -2, -1, 0, 1, 2, 3\}$

Ex. $\{x \in \mathbb{R} \mid |x| \leq 3\} = [-3, 3]$

Picture



Defn

A set S is called <u>finite</u> iff it contains exactly n elements for some non-negative integer n.
If S is not finite, it's called <u>infinite</u>.

<u>notation</u>

$$\mathbb{Z}_{\geq 0} = \{0, 1, 2, \ldots\}$$

$$\mathbb{Z}_{<0} = \{-1, -2, \ldots\}$$

$$\mathbb{Z}_{\geq 6} = \{6, 7, 8, \ldots\}$$

<u>notation</u>. $|S| = n$ means S is a finite set with n elements. We call $|S|$ the <u>cardinality</u> of S.

Also $|S| < \infty$ means S is finite

and $|S| = \infty$ means S is infinite

<u>(1.2) Cartesian products</u>

An ordered collection of n objects is called an <u>ordered n-tuple</u>.

<u>Notation:</u> $(x_1, x_2, x_3, \ldots, x_n)$

if $n = 2$, called an <u>ordered Pair</u>

<u>Note:</u> $(x_1, x_2) = (y_1, y_2)$ iff both $x_1 = y_1$ and $x_2 = y_2$.

<u>Ex.</u> $(1, 1, 2)$, repetition is allowed.

<u>Defn</u>

The <u>Cartesian Product</u> of two sets A, B is the set of all ordered pairs (x,y) with $x \in A$ and $y \in B$.

notation: $A \times B$

$$A \times B = \{(x,y) \mid x \in A \text{ and } y \in B\}$$

<u>Ex.</u> let $A = \{1,2\}$, $B = \{a,b,c\}$

$$A \times B = \{(1,a),(1,b),(1,c),(2,a),(2,b),(2,c)\}$$

$$B \times A = \{(a,1),(a,2),(b,1),(b,2),(c,1),(c,2)\}$$

Theorem

If A, B are finite sets, then $A \times B$ is also finite, and

$$|A \times B| = |A| \cdot |B|$$

Defn

The Cartesian Product of n sets $A_1, A_2, \ldots, A_n$ is the set

$$A_1 \times A_2 \times \cdots \times A_n$$
$$= \{ (x_1, x_2, \ldots, x_n) \mid x_i \in A_i \text{ for } 1 \leq i \leq n \}$$

Theorem

If $A_1, \ldots, A_n$ are finite, then

$$|A_1 \times A_2 \times \cdots \times A_n| = |A_1| \cdot |A_2| \cdot \cdots \cdot |A_n|$$

Exercises:

explain why $A \times B = B \times A$ iff $A = B$.

(1.3) Subsets

Defn

we say a set A is a subset of a set B iff every element of A is also an element of B.

notation: $A \subseteq B$

We say A is a proper subset of B iff $A \subseteq B$ but $A \neq B$

notation: $A \subsetneqq B$

other notation: $A \subset B$
we don't use it !!

observe for any set S

- $S \subseteq S$
- $\phi \subseteq S$

<u>Ex.</u> $\{2,3\} \subseteq \{1,2,3,4\}$.

also $\{2,3\} \subsetneq \{1,2,3,4\}$

(1.4) Power set of a set

<u>Defn</u>

The set of all subsets of a set S is called the <u>Power set</u> of S.

<u>notation</u>: $\mathbb{P}(S)$

Ex let $S = \{1, 2, 3\}$, $|S| = 3$

$$\mathbb{P}(S) = \{ \emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\} \}$$

observe: $|\mathbb{P}(S)| = 8 = 2^3 = 2^{|S|}$

Theorem

If S is finite, so is $\mathbb{P}(S)$ and

$$|\mathbb{P}(S)| = 2^{|S|}$$