CSE 16
Summer 2021
Quiz 3 **Solutions**

1. (20 Points) In a group of 35 people, all of whom drink coffee or tea (or both), 18 drink coffee and 12 drink both coffee and tea.
   a. (10 Points) How many in the group drink tea?

   Solution:
   Let $A = \{\text{coffee drinkers}\}$ and $B = \{\text{tea drinkers}\}$. By the principle of inclusion-exclusion, we have $|A \cup B| = |A| + |B| - |A \cap B|$. The given information says $|A \cup B| = 35$, $|A| = 18$ and $|A \cap B| = 12$. Therefore $35 = 18 + |B| - 12$, whence $|B| = 29$. Thus **29 people drink tea**. ■

   b. (10 Points) How many drink only tea (i.e. tea but not coffee)?

   Solution:
   Using the same notation as in (a), we have $|B - A| = |B - (A \cap B)| = 29 - 12 = 17$. Therefore **17 people drink only tea**. ■

2. (15 Points) Determine the coefficient of x^{-4} in the full expansion of $\left(x + \frac{1}{x}\right)^{10}$.

   Solution:
   The Binomial Theorem gives

$$\left(x + \frac{1}{x}\right)^{10} = \sum_{k=0}^{10} \binom{10}{k} \cdot x^{10-k} \cdot \left(\frac{1}{x}\right)^{k} = \sum_{k=0}^{10} \binom{10}{k} \cdot x^{10-2k}$$

   Setting $10 - 2k = -4$, we obtain $2k = 14 \Rightarrow k = 7$. Therefore the coefficient of x^{-4} is

$$\binom{10}{7} = \frac{10 \cdot 9 \cdot 8}{3 \cdot 2} = 10 \cdot 3 \cdot 4 = \mathbf{120}$$

■

3. (15 Points) Determine the number of strings of length 10 from the alphabet $\{A, B, C\}$ that contain exactly 3 A's and 4 B's.

   Solution:
   Each such string must contain $10 - 3 - 4 = 3$ C's, and therefore is an anagram of $AAABBBBCCC$. Equivalently, they are permutations of the multiset $[A, A, A, B, B, B, B, C, C, C]$. By a theorem proved in class, we have

$$(\#\text{strings}) = \frac{10!}{3! \cdot 4! \cdot 3!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 3 \cdot 2} = 10 \cdot 3 \cdot 4 \cdot 7 \cdot 5 = \mathbf{4,200}$$

■