CSE 16
Summer 2021
Quiz 2 **Solutions**

1. (20 Points) Let $K(x, y)$ be the statement "x knows y", where the universe of discourse for both x and y is the set of all people. (Assume it is possible for x to know y while y does not know x, and hence $K(x, y)$ and $K(y, x)$ are different statements.) Translate the sentence "there are two distinct persons who know each other, and no one else" into First Order Logic.

   Solution: $\exists x\, \exists y \left(x \neq y \wedge K(x, y) \wedge K(y, x) \wedge \forall z \left(\sim K(x, z) \wedge \sim K(y, z) \right) \right)$ ■

2. (15 Points) Let the universe of discourse for the following propositional functions be $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$, and define $P(n) = (\exists k : n = 3k)$ and $Q(n) = (\exists k : n = 5k + 3)$. Determine the 4 *smallest* elements in the set $\{ n \in \mathbb{N} \mid P(n) \wedge Q(n) \}$.

   Solution:
   Let $A = \{ n \in \mathbb{N} \mid P(n) \}$ and $B = \{ n \in \mathbb{N} \mid Q(n) \}$. Then

   $$A = \{ 0, \mathbf{3}, 6, 9, 12, 15, \mathbf{18}, 21, 24, 27, 30, \mathbf{33}, 36, 39, 42, 45, \mathbf{48}, 51, 54, 57, 60, \mathbf{63}, \ldots \ldots \}$$

   and

   $$B = \{ \mathbf{3}, 8, 13, \mathbf{18}, 23, 28, \mathbf{33}, 38, 43, \mathbf{48}, 53, 58, \mathbf{63}, 68, \ldots \ldots \}.$$

   Thus

   $$\{ n \in \mathbb{N} \mid P(n) \wedge Q(n) \} = A \cap B = \{ 3, 18, 33, 48, 63, \ldots \},$$

   whose 4 smallest elements are: **3**, **18**, **33**, **48**. ■

3. (15 Points) A banking system requires Personal Identification Numbers (PINs) that satisfy the following constraints.

   (1) Must consist of only decimal digits $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$.
   (2) Must have length at least 5, and at most 8.
   (3) Must not be the customer's birth date. (Here, a birth date would be encoded as a 6 digit number, so for instance May 3, 1998 would be 050398)

   How many different valid PINs are there?

   Solution:
   We partition the PINs satisfying (1) and (2) into 4 sets. By the multiplication principle we have

   length 5: # of PINs = 10^5
   length 6: # of PINs = 10^6
   length 7: # of PINs = 10^7
   length 8: # of PINs = 10^8

   There is only one PIN violating condition (3), so by the addition and subtraction principles, the number of valid PINs is: $10^5 + 10^6 + 10^7 + 10^8 - 1 = \mathbf{111{,}099{,}999}$. ■