

CSE 16
Summer 2021
Quiz 1

Solutions

1. (20 Points) Let $A \subseteq U$ and $B \subseteq U$. Suppose $|A \cap B| = 8$, $|A| = 10$, $|B| = 15$ and $|U| = 20$. Determine

$$|\mathbb{P}(\mathbb{P}(\overline{A \cup B}))|$$

Solution:

$$|A \cup B| = |A| + |B| - |A \cap B| = 10 + 15 - 8 = 17$$

$$|\overline{A \cup B}| = 20 - 17 = 3$$

$$|\mathbb{P}(\overline{A \cup B})| = 2^3 = 8$$

$$|\mathbb{P}(\mathbb{P}(\overline{A \cup B}))| = 2^8 = \mathbf{256}$$

2. (15 Points) Let $S = \{1, 2, 3, 4\}$. Determine the set $\{X \in \mathbb{P}(S) : |X| = 2\}$.

Solution:

We seek the set of 2-element subsets of S : $\{\{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}\}$

3. (15 Points) Determine whether the logical expression $(P \oplus Q) \vee (Q \Rightarrow \sim R)$ is a tautology. Either prove that it is a tautology (by any method), or show that it is not by finding an assignment of truth values to the propositional variables P, Q, R that makes the expression false.

Solution:

We construct a truth table for the expression.

P	Q	R	$P \oplus Q$	$Q \Rightarrow \sim R$	$(P \oplus Q) \vee (Q \Rightarrow \sim R)$
0	0	0	0	1	1
0	0	1	0	1	1
0	1	0	1	1	1
0	1	1	1	0	1
1	0	0	1	1	1
1	0	1	1	1	1
1	1	0	0	1	1
1	1	1	0	0	0

We conclude that $(P \oplus Q) \vee (Q \Rightarrow \sim R)$ is **not a tautology**, since the assignment $P = Q = R = T$ makes it false.