

CSE 16

Homework Assignment 2

Solutions to Selected Problems

(2.5) 10

Suppose the statement $((P \wedge Q) \vee R) \Rightarrow (R \vee S)$ is false. Find the truth values of P , Q , R and S . (Note this can be done without constructing a truth table.)

Solution:

The only way for an implication to be false is for the hypothesis to be true while the conclusion is false. Thus $(P \wedge Q) \vee R$ is true, and $R \vee S$ is false. Since $R \vee S$ is false, both operands must be false, i.e.

$$R = S = F.$$

Since R is false and $(P \wedge Q) \vee R$ is true, we must have $P \wedge Q$ true, hence both operands are true, i.e.

$$P = Q = T.$$

Thus if $((P \wedge Q) \vee R) \Rightarrow (R \vee S)$ is false, then **$P = Q = T$, and $R = S = F$** . ■

(2.6) 10

Decide whether $(P \Rightarrow Q) \vee R$ and $\sim((P \wedge \sim Q) \wedge \sim R)$ are logically equivalent.

P	Q	R	$P \Rightarrow Q$	$(P \Rightarrow Q) \vee R$	$\sim Q$	$P \wedge \sim Q$	$\sim R$	$(P \wedge \sim Q) \wedge \sim R$	$\sim((P \wedge \sim Q) \wedge \sim R)$
0	0	0	1	1	1	0	1	0	1
0	0	1	1	1	1	0	0	0	1
0	1	0	1	1	0	0	1	0	1
0	1	1	1	1	0	0	0	0	1
1	0	0	0	0	1	1	1	1	0
1	0	1	0	1	1	1	0	0	1
1	1	0	1	1	0	0	1	0	1
1	1	1	1	1	0	0	0	0	1

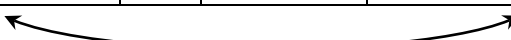
Since the two expressions have the same truth table, **they are logically equivalent**. ■

(2.6) 14

Decide if $P \wedge (Q \vee (\sim Q))$ and $(\sim P) \Rightarrow (Q \wedge (\sim Q))$ are logically equivalent.

Solution (truth table):

P	Q	$\sim Q$	$Q \vee (\sim Q)$	$P \wedge (Q \vee (\sim Q))$	$\sim P$	$Q \wedge (\sim Q)$	$(\sim P) \Rightarrow (Q \wedge (\sim Q))$
0	0	1	1	0	1	0	0
0	1	0	1	0	1	0	0
1	0	1	1	1	0	0	1
1	1	0	1	1	0	0	1



same

Since the truth tables of the two expressions are identical, **they are logically equivalent.** ■

Alternate Solution (using logical identities):

$$\begin{aligned}
 P \wedge (Q \vee (\sim Q)) &= P \wedge T && \text{(Negation law)} \\
 &= P && \text{(Identity law)}
 \end{aligned}$$

and

$$\begin{aligned}
 (\sim P) \Rightarrow (Q \wedge (\sim Q)) &= \sim (\sim P) \vee (Q \wedge (\sim Q)) && \text{(Implication law)} \\
 &= P \vee (Q \wedge (\sim Q)) && \text{(Double Negation law)} \\
 &= P \vee F && \text{(Negation law)} \\
 &= P && \text{(Identity law)}
 \end{aligned}$$

Since the two expressions are both logically equivalent to P , **they are equivalent to each other.** ■

(2.7) 10

Translate $\exists m \in \mathbb{Z}, \forall n \in \mathbb{Z}, m = n + 5$ into natural language, and determine whether it is true or false.

Solution:

Either one of the following translations work.

"There exists an integer m such that for all integers n , $m = n + 5$ "

or

"There is an integer that is 5 more than every integer"

This statement is **false**. ■