**CSE 16**
**Homework Assignment 2**
**Solutions to Selected Problems**

<u>**(2.5) 10**</u>

Suppose the statement $\big((P \wedge Q) \vee R\big) \Rightarrow (R \vee S)$ is false. Find the truth values of $P$, $Q$, $R$ and $S$. (Note this can be done without constructing a truth table.)

**Solution:**

The only way for an implication to be false is for the hypothesis to be true while the conclusion is false. Thus $(P \wedge Q) \vee R$ is true, and $R \vee S$ is false. Since $R \vee S$ is false, both operands must be false, i.e.

$$R = S = F.$$

Since $R$ is false and $(P \wedge Q) \vee R$ is true, we must have $P \wedge Q$ true, hence both operands are true, i.e.

$$P = Q = T.$$

Thus if $\big((P \wedge Q) \vee R\big) \Rightarrow (R \vee S)$ is false, then $\boldsymbol{P = Q = T}$**, and** $\boldsymbol{R = S = F}$. ■

<u>**(2.6) 10**</u>

Decide whether $(P \Rightarrow Q) \vee R$ and $\sim \big((P \wedge \sim Q) \wedge \sim R\big)$ are logically equivalent.

| $P$ | $Q$ | $R$ | $P \Rightarrow Q$ | $(P \Rightarrow Q) \vee R$ | $\sim Q$ | $P \wedge \sim Q$ | $\sim R$ | $(P \wedge \sim Q) \wedge \sim R$ | $\sim \big((P \wedge \sim Q) \wedge \sim R\big)$ |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 |

Since the two expressions have the same truth table, **they are logically equivalent**. ■

**(2.6) 14**

Decide if $P \wedge (Q \vee (\sim Q))$ and $(\sim P) \Rightarrow (Q \wedge (\sim Q))$ are logically equivalent.

**Solution (truth table):**

| $P$ | $Q$ | $\sim Q$ | $Q \vee (\sim Q)$ | $P \wedge (Q \vee (\sim Q))$ | $\sim P$ | $Q \wedge (\sim Q)$ | $(\sim P) \Rightarrow (Q \wedge (\sim Q))$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 0 | 1 | 1 | 0 | 0 | 1 |

same

Since the truth tables of the two expressions are identical, **they are logically equivalent**. ■

**Alternate Solution (using logical identities):**

$$\begin{aligned} P \wedge (Q \vee (\sim Q)) &= P \wedge T && \text{(Negation law)} \\ &= P && \text{(Identity law)} \end{aligned}$$

and

$$\begin{aligned} (\sim P) \Rightarrow (Q \wedge (\sim Q)) &= \sim (\sim P) \vee (Q \wedge (\sim Q)) && \text{(Implication law)} \\ &= P \vee (Q \wedge (\sim Q)) && \text{(Double Negation law)} \\ &= P \vee F && \text{(Negation law)} \\ &= P && \text{(Identity law)} \end{aligned}$$

Since the two expressions are both logically equivalent to $P$, **they are equivalent to each other**. ■

**(2.7) 10**

Translate $\exists m \in \mathbb{Z}, \forall n \in \mathbb{Z}, m = n + 5$ into natural language, and determine whether it is true or false.

**Solution:**

Either one of the following translations work.

**"There exists an integer $m$ such that for all integers $n$, $m = n + 5$"**

or

**"There is an integer that is 5 more than every integer"**

This statement is **false**. ■