

CSE 16

Homework Assignment 1

Solutions to Selected Problems

(1.2) 2g

Let $A = \{\pi, e, 0\}$ and $B = \{0, 1\}$. Then

$$B \times B = \{ (0,0), (0,1), (1,0), (1,1) \}$$

and

$$A \times (B \times B) = \{ (\pi, (0,0)), (\pi, (0,1)), (\pi, (1,0)), (\pi, (1,1)), \\ (e, (0,0)), (e, (0,1)), (e, (1,0)), (e, (1,1)), \\ (0, (0,0)), (0, (0,1)), (0, (1,0)), (0, (1,1)) \}$$

(1.3) 8

Let $S = \{ \{0, 1\}, \{0, 1, \{2\}\}, \{0\} \}$. Then

$$\mathbb{P}(S) = \{ \emptyset, \\ \{\{0, 1\}\}, \quad \{\{0, 1, \{2\}\}\}, \quad \{\{0\}\}, \\ \{\{0, 1\}, \{0, 1, \{2\}\}\}, \quad \{\{0, 1\}, \{0\}\}, \quad \{\{0, 1, \{2\}\}, \{0\}\}, \\ \{\{0, 1\}, \{0, 1, \{2\}\}, \{0\}\} \\ \}$$

(1.4) 20

Let $|A| = m$ so that $|\mathbb{P}(A)| = 2^m$. The number of 1-element subsets of $\mathbb{P}(A)$ is therefore also 2^m . There is only one 0-element subset of $\mathbb{P}(A)$, namely $\emptyset$. Thus

$$\{ X \subseteq \mathbb{P}(A) : |X| \leq 1 \} = \{ X \subseteq \mathbb{P}(A) : |X| = 1 \} \cup \{ X \subseteq \mathbb{P}(A) : |X| = 0 \}$$

and therefore

$$|\{ X \subseteq \mathbb{P}(A) : |X| \leq 1 \}| = |\{ X \subseteq \mathbb{P}(A) : |X| = 1 \}| + |\{ X \subseteq \mathbb{P}(A) : |X| = 0 \}| \\ = 2^m + 1$$

(1.8) 14

For each $\alpha \in I$, let A_α be a set. Suppose that $\emptyset \neq J \subseteq I$. It follows that

$$\bigcap_{\alpha \in I} A_\alpha \subseteq \bigcap_{\alpha \in J} A_\alpha$$

Explanation:

Let x be an element of the left hand side (LHS). Then $x \in A_\alpha$ for all $\alpha \in I$. Since $J \subseteq I$, this implies $x \in A_\alpha$ for all $\alpha \in J$. Therefore x is also a member of the right hand side (RHS), and hence $\text{LHS} \subseteq \text{RHS}$. ■